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Probing the seesaw mechanism with neutrino data and leptogenesis

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Abstract: In the framework of the seesaw mechanism with three heavy right-handed Majorana neutrinos and no Higgs triplets we carry out a systematic study of the structure of the right-handed neutrino sector. Using the current low-energy neutrino data as an input and assuming hierarchical Dirac-type neutrino masses m_{Di}, we calculate the masses M_i and the mixing of the heavy neutrinos. We confront the inferred properties of these neutrinos with the constraints coming from the requirement of a successful baryogenesis via leptogenesis. In the generic case the masses of the right-handed neutrinos are highly hierarchical: $M_i \propto m_{Di}^2$; the lightest mass is $M_1 \approx 10^3 - 10^6$ GeV and the generated baryon-to-photon ratio $\eta_B \lesssim 10^{-14}$ is much smaller than the observed value. We find the special cases which correspond to the level crossing points, with maximal mixing between two quasi-degenerate right-handed neutrinos. Two level crossing conditions are obtained: $m_{ee} \approx 0$ (1-2 crossing) and $d_{12} \approx 0$ (2-3 crossing), where m_{ee} and d_{12} are respectively the 11-entry and the 12-subdeterminant of the light neutrino mass matrix in the basis where the neutrino Yukawa couplings are diagonal. We show that sufficient lepton asymmetry can be produced only in the 1-2 crossing where $M_1 \approx M_2 \approx 10^8$ GeV, $M_3 \approx 10^{14}$ GeV and $(M_2 - M_1)/M_2 \lesssim 10^{-5}$.

Keywords: Baryogenesis, Neutrino Physics.

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Contents

1. Introduction

The seesaw mechanism of neutrino mass generation [1] provides a very simple and appealing explanation of the smallness of the neutrino mass. The low-energy neutrino mass matrix m is given in terms of the Majorana mass matrix of the right-handed (RH) neutrinos, M_R, and the Dirac mass matrix, m_D, as

$$m = -m_D M_R^{-1} m_D^T \,. \tag{1.1}$$

While the elements of m_D are expected to be at or below the EW scale, the characteristic mass scale of RH neutrinos is naturally the GUT or parity breaking scale. A very important feature of the seesaw mechanism, which makes it even more attractive, is that it has a simple and elegant built-in mechanism of production of the baryon asymmetry of the Universe – baryogenesis through leptogenesis [2].

The main prediction of the seesaw mechanism is the existence of RH Majorana neutrinos N_i. Being extremely heavy, these neutrinos are not accessible to direct experimental studies, though several indirect ways of probing the properties of the RH sector are known:

- studies of leptogenesis [3];

- searches for signatures of Grand Unification;
- renormalization group running effects induced by RH neutrinos (e.g., on the $b-\tau$ unification [4]).

What can be learned about the heavy RH neutrino sector, using the currently available low-energy neutrino data as an input? The matrix m in (1.1) can to a large extent be reconstructed from the available low-energy data; then eq. (1.1) can be used to study M_R.

Obviously, for such an analysis one would also need to know the Dirac neutrino mass matrix m_D. Unfortunately, at present no information on m_D is available, since there are almost no ways of studying it experimentally (though, in the context of certain SUSY models, some information on m_D can be obtained from the studies of the rare decays like $\mu \to e\gamma$, $\tau \to e\gamma$ [5][1]). One is, therefore, forced to resort to some theoretical arguments. Such arguments are in general provided by the GUTs, in which m_D is typically related at the unification scale to the mass matrices of up-type or down-type quarks or of charged leptons. Since all the quark and charged lepton masses are highly hierarchical, we will assume this to be true also for m_D. Moreover, in models in which m_D is related to the up-type quark mass matrices and the mass matrix of charged leptons m_l, to the down-type quark matrix, one can expect that the left-handed rotations diagonalizing m_l and m_D are nearly the same. Their mismatch (described by the matrix U_L) is expected not to exceed the Cabibbo mixing in the quark sector ($U_L \sim U_{CKM}$). The large leptonic mixing observed in the low-energy sector is then a consequence of the "seesaw enhancement of lepton mixing" [7, 8].[2] Such an enhancement requires a strong (quadratic) mass hierarchy of RH neutrinos and/or off-diagonal structure of M_R.

In this framework, studies of the structure of RH sector and leptogenesis have been performed in a number of publications [10, 11]. It was realized that, due to a strong hierarchy of neutrino Dirac Yukawa couplings (analogous to that of up-type quarks), the predicted baryon asymmetry is much smaller than the observed one, especially [12, 13] in the case of the LMA MSW solution of the solar neutrino problem. The asymmetry can be much larger if the hierarchy of Yukawa couplings is similar to that of the down quarks or charged leptons and the largest coupling is of order 1 [14]. In this case the hierarchy between the masses of RH neutrinos becomes weaker and, furthermore, large RH mixing can appear.

In this paper, we follow a bottom-up approach to reconstruct the RH neutrino sector, under the assumption of hierarchical Dirac masses. We perform a systematic study of all possible structures of the RH neutrino mass matrix consistent with the low energy neutrino data. Although our general formalism is valid for an arbitrary left-handed Dirac-type mixing, in most of our quantitative analysis we assume this mixing to be small; we comment on the opposite situation in the last section. We study the dependence of the

[1]Using constraints coming from these lepton flavor violating decays together with neutrino data, one can, in principle, reconstruct the mass matrices relevant for the seesaw. The consequences for leptogenesis are discussed, e.g., in [6].

[2]There also exists a different class of seesaw models in which the large low-energy leptonic mixing is due to the large left-handed Dirac-type mixing [9].

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produced lepton asymmetry on the structure of M_R, calculating explicitly the RH mixing matrix U_R and the relevant CP-violating phases. We confirm that, in the generic case, too small an asymmetry is produced. We identify and study in detail the special cases in which the observed baryon asymmetry can be generated.

The paper is organized as follows. In section 2 we formulate our framework. In section 3 we discuss the generic case when the three RH neutrinos have strongly hierarchical masses. We give simple analytic expressions for the masses and the mixings of the RH neutrinos. In section 4 we describe the conditions under which the special cases are realized. They correspond to partial or complete degeneracy of the RH neutrinos. In section 5 we analyze the special case which leads to a successful leptogenesis. In sections 6 and 7 other special cases are described. Section 8 contains discussion of our results and conclusions.

2. The framework

2.1 Low energy data and structure of light neutrino mass matrix

In the flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$ the Majorana mass matrix of light neutrinos, m, can be written in terms of the observables as

$$m = U^*_{PMNS} m^{\rm diag} U^\dagger_{PMNS} \,, \tag{2.1}$$

where $m^{\rm diag} \equiv diag(m_1,\ m_2,\ m_3)$ and U_{PMNS} [15] is the leptonic mixing matrix, for which we use the standard parameterization:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot K_0 \,. \tag{2.2}$$

Here $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, δ is the CP-violating Dirac phase and

$$K_0 = {\rm diag}(e^{i\rho}, 1, e^{i\sigma})$$

contains the two CP-violating Majorana phases. Except in a few cases, we will absorb ρ and σ in the light neutrino masses m_i, which are therefore allowed to be complex.

From the solar, atmospheric, accelerator and reactor neutrino experiments we take the following input (at 90% C.L.) [16]:

$$\begin{aligned} &\Delta m^2_{\rm sol} \equiv \Delta m^2_{12} = \left(7\ ^{+10}_{-2}\right) \cdot 10^{-5}\,{\rm eV}^2\,, && \tan^2\theta_{12} = 0.42^{+0.2}_{-0.1}\,; \\ &\Delta m^2_{\rm atm} \equiv \Delta m^2_{23} = \left(2.5\ ^{+1.4}_{-0.9}\right) \cdot 10^{-3}\,{\rm eV}^2\,, && \tan\theta_{23} = 1\ ^{+0.35}_{-0.25}; \\ &\sin\theta_{13} \lesssim 0.2\,. \end{aligned} \tag{2.3}$$

The neutrinoless 2β decay experiments [17] restrict the ee-element of the matrix m:

$$|m_{ee}| < (0.35 \div 1.3)\,{\rm eV} \qquad (90\%\ {\rm C.L.})\,.$$

The direct kinematic measurements give an upper bound on neutrino masses, relevant in the case of degenerate mass spectrum: $m_{\nu_e} \approx |m_i| < 2.2\,{\rm eV}$ [18]. However, the cosmological bound on the sum of light neutrino masses (at 95% C.L.) [19, 20],

$$\sum\nolimits_i |m_i| < (0.7 \div 2.1)\,{\rm eV}\,, \tag{2.4}$$

leads to the strongest limit on individual masses: $|m_i| < (0.23 \div 0.70)\,{\rm eV}$.

Using this phenomenological information one can, to a large extent, reconstruct the mass matrix m, just by inserting the data (2.3) and (2.4) into eq. (2.1) [21]–[23]. A significant freedom still exists due to the unknown absolute mass scale $|m_1|$ and CP-violating phases ρ, σ. The dependence of the structure of m on the unknown s_{13} and δ is weaker because of the smallness of s_{13}.

In spite of the above-mentioned freedom, a generic feature of the mass matrix m emerges: all its elements are of the same order (within a factor of 10 or so of each other), except in some special cases. The reason for this is twofold:

- the two large mixing angles θ_{12} and θ_{23};
- a relatively weak hierarchy between the mass eigenvalues: according to the LMA MSW solution of the solar neutrino problem,

$$\frac{|m_2|}{|m_3|} \geq \sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} > 0.1 \div 0.15\,. \tag{2.5}$$

We will refer to the situation when all the matrix elements of m are of the same order and there are no special cancellations as the generic case. The "quasi-democratic" structure of the mass matrix m is the main starting point of our analysis; it has important implications for the seesaw mechanism and leptogenesis, as we will see in section 3.

A strong hierarchy between certain matrix elements of m can be realized only for specific values of the absolute mass scale and CP-violating phases,[3] and we will consider these special cases separately (sections 5–7).

2.2 Dirac mass matrix

In the basis where the mass matrix of RH Majorana neutrinos is diagonal, the Dirac mass matrix can be written as

$$m_D = U_L^\dagger m_D^{\rm diag} U_R\,. \tag{2.6}$$

Here U_L and U_R are unitary matrices and $m_D^{\rm diag} \equiv diag(m_u, m_c, m_t)$, with the mass eigenvalues $m_{u,c,t}$ being real and positive. We have denoted the eigenvalues of m_D in analogy with up-type quark masses, but we do not require the exact coincidence of the quark and leptonic masses. Our main assumption in this paper is that there is a strong hierarchy of the eigenvalues of m_D:

$$m_u \ll m_c \ll m_t\,, \tag{2.7}$$

similar to the hierarchy of the quark or charged lepton masses. For numerical estimates, we will use the reference values

$$m_u = 1\,{\rm MeV}\,, \qquad m_c = 400\,{\rm MeV}\,, \qquad m_t = 100\,{\rm GeV}\,, \tag{2.8}$$

which approximately coincide with the up-type quark masses at the mass scale $\sim 10^9$ GeV [26].

[3] Realistic neutrino mass textures with some exact zeros have been studied, e.g., in [24]. Also the seesaw realizations of these zeros have been considered [25].

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The matrix U_L defined in eq. (2.6) describes the mismatch between the left-handed rotations diagonalizing the charged lepton and neutrino Dirac mass matrices and, therefore, is the leptonic analogue of the quark CKM mixing matrix. It differs from the leptonic mixing matrix U_{PMNS}, defined in eq. (2.1), which is probed in the low-energy neutrino experiments. The difference between U_L and U_{PMNS} is a consequence of the seesaw mechanism. By analogy with the CKM matrix where the mixing is small, one expects that the matrix U_L is close to the unit matrix.

2.3 Conditions for a successful leptogenesis

Let us consider the constraints on the seesaw parameters coming from the requirement of the successful thermal leptogenesis. We assume that a lepton asymmetry is generated by the CP-violating out-of-equilibrium decays of RH neutrinos [2]. The lepton asymmetry is then converted to a baryon asymmetry through the sphaleron processes [27], thus explaining the baryon asymmetry of the Universe. We will use the recent experimental value of the baryon-to-photon ratio [19],

$$\eta_B = \left(6.5 \pm^{0.4}_{0.3}\right) \cdot 10^{-10}\,. \tag{2.9}$$

The lepton number asymmetry, ϵ_i, produced in the decay of a RH neutrino with the mass $|M_i|$ can be written as [28]:

$$\epsilon_i = \frac{1}{8\pi} \sum_{k \neq i} f\left(\frac{|M_k|^2}{|M_i|^2}\right) \frac{\mathrm{Im}[(h^\dagger h)^2_{ik}]}{(h^\dagger h)_{ii}}\,. \tag{2.10}$$

Here h is the matrix of neutrino Yukawa couplings in the basis where M_R is diagonal with real and positive eigenvalues. Using the relation $h \equiv m_D/v$ (where $v = 174\,\mathrm{GeV}$ is the electroweak VEV) and m_D given in (2.6) we can write[4]

$$h^\dagger h = \frac{1}{v^2} U_R^\dagger (m_D^{\mathrm{diag}})^2 U_R\,. \tag{2.11}$$

We note in passing that in general we allow the non-zero elements M_i of the diagonalized RH neutrino mass matrix M_R^{diag} to be complex. However, the unitary matrix U_R in eq. (2.11) is defined in such a way that it relates the basis where m_D is diagonal to the basis where M_R is diagonal with real and positive non-zero elements, i.e. the phases of M_i should be included into the definition of U_R (cf. eqs. (3.6) and (3.7) below). We will be always assuming the ordering $|M_1| \leq |M_2| \leq |M_3|$.

In the standard electroweak model the function f in eq. (2.10) is given by

$$f(x) = \sqrt{x}\left[\frac{2-x}{1-x} - (1+x)\log\left(\frac{1+x}{x}\right)\right]. \tag{2.12}$$

This expression is valid for $||M_i| - |M_j|| \gg \Gamma_i + \Gamma_j$, where Γ_i is the decay width of the ith RH neutrino, given at tree level by

$$\Gamma_i = \frac{(h^\dagger h)_{ii}}{8\pi} |M_i|\,.$$

[4] In SUSY models v should be replaced with $v \sin\beta$. For $\tan\beta \gtrsim 3$, this corresponds to a very small rescaling of Yukawa couplings.

In the limit of the quasi-degenerate neutrinos ($x = |M_j/M_i|^2 \to 1$), one formally obtains from (2.12)

$$f(x) \approx \frac{1}{1-x} \approx \frac{|M_i|}{2(|M_i| - |M_j|)} \to \infty\,. \tag{2.13}$$

However, in reality the enhancement of the asymmetry is limited by the decay widths Γ_i and is maximized when $||M_i| - |M_j|| \sim \Gamma_i + \Gamma_j$ [29].

The left-handed rotation U_L does not enter the expression for the lepton number asymmetry. Furthermore, $h^\dagger h$ is invariant under the transformation

$$U_R \to D U_R\,, \tag{2.14}$$

where

$$D = \mathrm{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}) \tag{2.15}$$

and α, β, γ are arbitrary phases. Consequently, all the phases that can be removed from U_R by the transformation (2.14) have no impact on leptogenesis.

The baryon-to-photon ratio can be written as [30]

$$\eta_B \simeq 0.01 \sum_i \epsilon_i \cdot \kappa_i\,,$$

where the factors κ_i describe the washout of the produced lepton asymmetry ϵ_i due to various lepton number violating processes. In the domain of the parameter space which is of interest to us, they depend mainly on the effective mass parameters

$$\tilde{m}_i \equiv \frac{v^2 (h^\dagger h)_{ii}}{|M_i|} = \frac{[U_R^\dagger (m_D^{\mathrm{diag}})^2 U_R]_{ii}}{|M_i|}\,. \tag{2.16}$$

For $10^{-2}\,\mathrm{eV} < \tilde{m}_1 < 10^3\,\mathrm{eV}$, the washout factor κ_1 can be well approximated by [31]

$$\kappa_1(\tilde{m}_1) \simeq 0.3 \left(\frac{10^{-3}\,\mathrm{eV}}{\tilde{m}_1}\right) \left(\log \frac{\tilde{m}_1}{10^{-3}\,\mathrm{eV}}\right)^{-0.6}\,. \tag{2.17}$$

When $|M_1| \ll |M_{2,3}|$, only the decays of the lightest RH neutrino N_1 are relevant for producing the baryon asymmetry η_B, since the lepton asymmetry generated in the decays of the heavier RH neutrinos is washed out by the L-violating processes involving N_1's, which are very abundant at high temperatures $T \sim |M_{2,3}|$. At the same time, at $T \sim |M_1|$ the heavier neutrinos N_2 and N_3 have already decayed and so cannot wash out the asymmetry produced in the decays of N_1.

In refs. [32, 30], under the assumption $|M_1| \ll |M_{2,3}|$, the following absolute lower bound on the mass of the lightest RH neutrino was found from the condition of the successful leptogenesis:

$$|M_1| \gtrsim 4 \cdot 10^8\,\mathrm{GeV}\,. \tag{2.18}$$

The bound (2.18) corresponds to $\tilde{m}_1 \to 0$ and maximal ϵ_1; for other values of $\tilde{m}_1$ and ϵ_1 it is even stronger [30, 33]. Inequality (2.18) has been derived before the latest WMAP data became available [19]. These data (see eq. (2.9)) strengthen the bound by a factor ~ 1.5: $|M_1| \gtrsim 6 \cdot 10^8\,\mathrm{GeV}$.

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2.4 Mass matrix of RH neutrinos

Using the representation (2.6) for m_D, the matrix of light neutrinos can be written as

$$m = -U_L^\dagger m_D^{\rm diag} U_R (M_R^{\rm diag})^{-1} U_R^T m_D^{\rm diag} U_L^* \,.$$

Then, in the basis where

$$m_D = U_L^\dagger m_D^{\rm diag} \,, \tag{2.19}$$

the inverse mass matrix of the RH neutrinos equals

$$W \equiv M_R^{-1} = U_R (M_R^{\rm diag})^{-1} U_R^T \,, \tag{2.20}$$

and, correspondingly, the matrix M_R itself is given by

$$M_R = U_R^* M_R^{\rm diag} U_R^\dagger \,. \tag{2.21}$$

From the seesaw formula one obtains, in the basis (2.19),

$$W = -m_D^{-1} m (m_D^{-1})^T = -(m_D^{\rm diag})^{-1} \hat{m} (m_D^{\rm diag})^{-1} \,, \tag{2.22}$$

where

$$\hat{m} \equiv U_L m U_L^T \,. \tag{2.23}$$

When $U_L = \mathbb{1}$, that is, the Dirac left-handed rotation is absent, one has $\hat{m} = m$. When U_L slightly deviates from $\mathbb{1}$ (e.g., $U_L \approx U_{CKM}$), the difference between $\hat{m}$ and m is within the present experimental uncertainty, apart from some particular cases. In most of our analysis we shall be assuming $U_L = \mathbb{1}$ and neglect the difference between $\hat{m}$ and m. All the analytic expressions that we derive for $U_L = \mathbb{1}$ are also valid for an arbitrary U_L, if one substitutes the matrix elements of m by the corresponding elements of $\hat{m}$.

In supersymmetric scenarios, U_L can be probed in lepton flavor violating decays like $\mu \to e\gamma$ or $\tau \to \mu\gamma$. If $U_L = \mathbb{1}$, these decays are strongly suppressed, whereas for $U_L \approx U_{CKM}$ one finds the predicted branching ratios to be close to the experimental upper bounds, provided that the slepton masses are of the order of $(100 \div 200)$ GeV and the neutrino Dirac masses take the values given in eq. (2.8). Therefore, if future experiments find a signal close to the present upper bounds, this will not require large left-handed rotations and so will not invalidate our results.

Using $m_D^{\rm diag} \equiv diag(m_u, m_c, m_t)$ in eq. (2.22) and taking $\hat{m} = m$, we obtain the following symmetric matrix W:

$$W = -\begin{pmatrix} \dfrac{m_{ee}}{m_u^2} & \dfrac{m_{e\mu}}{m_u m_c} & \dfrac{m_{e\tau}}{m_u m_t} \\ \cdots & \dfrac{m_{\mu\mu}}{m_c^2} & \dfrac{m_{\mu\tau}}{m_c m_t} \\ \cdots & \cdots & \dfrac{m_{\tau\tau}}{m_t^2} \end{pmatrix} . \tag{2.24}$$

In what follows we will find the eigenvalues of W and the mixing matrix U_R that diagonalizes W according to eq. (2.20).

3. The generic case

As discussed in section 2.1, in general the matrix elements $m_{\alpha\beta}$ are all of the same order of magnitude. We have defined this situation as the generic case. It follows then from (2.24) that the elements of W are highly hierarchical, with W_{11} being by far the largest one. Introducing for illustration the small expansion parameter

$$\lambda \sim \frac{m_u}{m_c} \sim \frac{m_c}{m_t} \sim 10^{-2}\,, \tag{3.1}$$

we obtain

$$W \sim -\frac{m_{ee}}{m_u^2}\begin{pmatrix} 1 & \lambda & \lambda^2 \\ \ldots & \lambda^2 & \lambda^3 \\ \ldots & \ldots & \lambda^4 \end{pmatrix}\,,$$

where in each element factors of order 1 are understood.

The largest eigenvalue of W is given, to a very good approximation, by the dominant (11)-element:

$$M_1 \approx \frac{1}{W_{11}} = -\frac{m_u^2}{m_{ee}}\,. \tag{3.2}$$

The second largest eigenvalue of W can be obtained from the dominant (12)-block of the matrix (2.24), just by dividing its determinant by W_{11}. The mass M_2 is then the inverse of this eigenvalue:

$$M_2 \approx \frac{W_{11}}{W_{11}W_{22} - W_{12}^2} = \frac{m_c^2 m_{ee}}{m_{e\mu}^2 - m_{ee}m_{\mu\mu}}\,. \tag{3.3}$$

The smallest eigenvalue of W can be found from the condition

$$(m_u m_c m_t)^2 = -m_1 m_2 m_3 M_1 M_2 M_3 \tag{3.4}$$

which is obtained by taking the determinants of both sides of eq. (1.1). This yields

$$M_3 \approx \frac{m_t^2(m_{e\mu}^2 - m_{ee}m_{\mu\mu})}{m_1 m_2 m_3}\,. \tag{3.5}$$

Thus, in the generic case the RH neutrinos have a very strong mass hierarchy: $M_1 \propto m_u^2$, $M_2 \propto m_c^2$, $M_3 \propto m_t^2$, in agreement with the "seesaw enhancement" condition [7].

The matrix W is diagonalized, to a high accuracy, by

$$U_R \approx \begin{pmatrix} 1 & -\left(\dfrac{m_{e\mu}}{m_{ee}}\right)^* \dfrac{m_u}{m_c} & \left(\dfrac{d_{23}}{d_{12}}\right)^* \dfrac{m_u}{m_t} \\ \left(\dfrac{m_{e\mu}}{m_{ee}}\right) \dfrac{m_u}{m_c} & 1 & -\left(\dfrac{d_{13}}{d_{12}}\right)^* \dfrac{m_c}{m_t} \\ \left(\dfrac{m_{e\tau}}{m_{ee}}\right) \dfrac{m_u}{m_t} & \left(\dfrac{d_{13}}{d_{12}}\right) \dfrac{m_c}{m_t} & 1 \end{pmatrix} \cdot K\,, \tag{3.6}$$

where

$$d_{23} \equiv m_{e\mu}m_{\mu\tau} - m_{\mu\mu}m_{e\tau}\,, \qquad d_{13} \equiv m_{ee}m_{\mu\tau} - m_{e\mu}m_{e\tau}\,, \qquad d_{12} \equiv m_{ee}m_{\mu\mu} - m_{e\mu}^2$$

and

$$K = \mathrm{diag}(e^{-i\phi_1/2}, e^{-i\phi_2/2}, e^{-i\phi_3/2})\,, \qquad \phi_i \equiv \arg M_i\,. \tag{3.7}$$

As can be seen from eq. (3.6), the RH mixing is very small in the generic case. We therefore encounter an apparently paradoxical situation, when both the left-handed and RH mixing angles are small and yet one arrives at a strong mixing in the low-energy sector. This is an example of the so-called "seesaw enhancement" of the leptonic mixing [7, 8]. The reason for this enhancement can be readily understood. Indeed, small mixing in m_D and W is related to the hierarchical structures of these matrices; however, in the seesaw formula (1.1) these hierarchies act in the opposite directions and largely compensate each other, leading to a "quasi-democratic" m and thus to large mixing in the low-energy sector.

The masses of the heavy neutrinos (eqs. (3.2), (3.3) and (3.5)) can be rewritten as functions of the low-energy observables using the expressions of $m_{\alpha\beta}$ in terms of the masses and mixing of light neutrinos [21]–[23]. In the limit $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, we find from eq. (2.1)

$$\begin{aligned} M_1 &= -\frac{m_u^2}{m_1 c_{12}^2 + m_2 s_{12}^2}\,, \\ M_2 &= -\frac{2m_c^2(m_1 c_{12}^2 + m_2 s_{12}^2)}{m_3(m_1 c_{12}^2 + m_2 s_{12}^2) + m_1 m_2}\,, \\ M_3 &= -\frac{m_t^2[m_3(m_1 c_{12}^2 + m_2 s_{12}^2) + m_1 m_2]}{2m_1 m_2 m_3}\,. \end{aligned} \tag{3.8}$$

The dependence of $|M_i|$ on $|m_1|$ is shown in figure 1. In the case of the normal hierarchy ($|m_1| \ll |m_{2,3}|$), these equalities take a particularly simple form (found previously in [13]):

$$M_1 \approx -\frac{m_u^2}{m_2 s_{12}^2}\,, \qquad M_2 \approx -\frac{2m_c^2}{m_3}\,, \qquad M_3 \approx -\frac{m_t^2 s_{12}^2}{2m_1}\,. \tag{3.9}$$

Notice that the lightest RH neutrino mass $|M_1|$ is related to the solar mass squared difference ($|m_2|^2 \approx \Delta m^2_{\rm sol}$), $|M_2|$ to the atmospheric one ($|m_3|^2 \approx \Delta m^2_{\rm atm}$), and $|M_3|$ is inversely proportional to m_1, for which we can use the upper bound $|m_1| < \sqrt{\Delta m^2_{\rm sol}}$. It is illuminating to rewrite eq. (3.9) in the "standard" seesaw form, expressing the light neutrino masses through the heavy neutrino ones:

$$m_1 \approx -\frac{m_t^2 s_{12}^2}{2M_3}\,, \qquad m_2 \approx -\frac{m_u^2}{s_{12}^2 M_1}\,, \qquad m_3 \approx -\frac{2m_c^2}{M_2}\,. \tag{3.10}$$

Comparing this with the naive seesaw expectations, we see that the expected correspondence between the masses of the light neutrinos and the Dirac masses ($m_1 \propto m_u^2$, $m_2 \propto m_c^2$, $m_3 \propto m_t^2$) is completely broken; this is due to the large neutrino mixing angles (in particular, to the fact that the solution of the solar neutrino problem is the LMA MSW one).

Numerically, from eq. (3.9) we find

$$|M_1| \simeq \frac{m_u^2}{s_{12}^2\sqrt{\Delta m^2_{\rm sol}}} \simeq 4.4\cdot 10^5\,\mathrm{GeV}\left(\frac{m_u}{1\,\mathrm{MeV}}\right)^2\,, \tag{3.11}$$

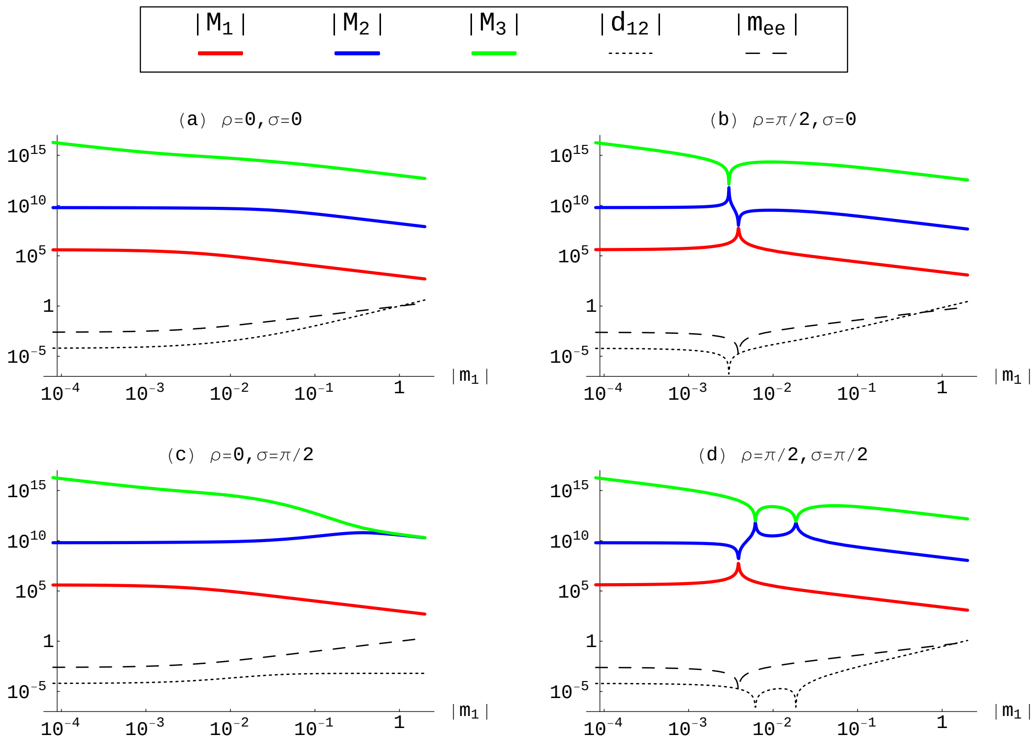

Figure 1: The masses of RH neutrinos $|M_i|$ in GeV as functions of the light neutrino mass $|m_1|$ in eV (solid thick lines), for different values of the Majorana phases of light neutrinos, ρ and σ. We have assumed normal mass ordering; $s_{13} = 0$; the best fit values of solar and atmospheric mixing angles and mass squared differences (eq. (2.3)); the values of Dirac-type neutrino masses $m_{u,c,t}$ given in eq. (2.8). Also shown are $|d_{12}| \equiv |m_{ee}m_{\mu\mu} - m_{e\mu}^2|$ in eV^2 (dotted thin line) and $|m_{ee}|$ in eV (dashed thin line) as functions of $|m_1|$.

$$|M_2| \simeq \frac{2m_c^2}{\sqrt{\Delta m^2_{\rm atm}}} \simeq 6.4 \cdot 10^9\,\mathrm{GeV}\left(\frac{m_c}{400\,\mathrm{MeV}}\right)^2, \tag{3.12}$$

$$|M_3| \simeq \frac{m_t^2 s_{12}^2}{2|m_1|} > 1.8 \cdot 10^{14}\,\mathrm{GeV}\left(\frac{m_t}{100\,\mathrm{GeV}}\right)^2. \tag{3.13}$$

These values of $|M_i|$ are illustrated by the leftmost regions (corresponding to $|m_1| \to 0$) of the plots in figure 1. For the inverted mass hierarchy ($|m_3| \ll |m_1| \simeq |m_2|$), from eq. (3.8) one finds

$$|M_1| \simeq (2-5) \cdot 10^4\,\mathrm{GeV}\left(\frac{m_u}{1\,\mathrm{MeV}}\right)^2, \tag{3.14}$$

$$|M_2| \simeq (3-6) \cdot 10^9\,\mathrm{GeV}\left(\frac{m_c}{400\,\mathrm{MeV}}\right)^2, \tag{3.15}$$

$$|M_3| > 10^{14}\,\mathrm{GeV}\left(\frac{m_t}{100\,\mathrm{GeV}}\right)^2. \tag{3.16}$$

Similar estimates hold true also in the quasi-degenerate case ($|m_1| \simeq |m_2| \simeq |m_3| \simeq m_0$), except that the inequality sign in eq. (3.16) has to be replaced by the approximate equality one and the right-hand sides of eqs. (3.14)–(3.16) have to be divided by $m_0/\sqrt{\Delta m^2_{\rm atm}} \approx 20 m_0/\,\mathrm{eV}$. In particular, for the lightest of the RH neutrinos we obtain

$$|M_1| \simeq (1 - 2.5)\,\mathrm{TeV}\left(\frac{m_u}{1\,\mathrm{MeV}}\right)^2\left(\frac{1\,\mathrm{eV}}{m_0}\right). \tag{3.17}$$

For the highest allowed by cosmological observations value, $m_0 = 0.7\,\mathrm{eV}$, eq. (3.17) gives $|M_1| \simeq (1.4 - 3.5)\,\mathrm{TeV}$. The values of $|M_i|$ in the quasi-degenerate case are shown on the right-hand side (corresponding to $|m_1| \approx m_0 \sim 1\,\mathrm{eV}$) of panels a, b, d in figure 1.

We now turn to the discussion of leptogenesis in the generic case under the consideration. Since the RH neutrino masses are highly hierarchical, the main contribution to the lepton asymmetry comes from the decays of the lightest RH neutrino N_1. From eq. (3.11) we find that, for $m_u \lesssim 10\,\mathrm{MeV}$, its mass is at least one order of magnitude smaller than the absolute lower bound (2.18). The normal hierarchy case is the most favorable one: for the other cases $|m_{ee}|$ is larger, leading to even smaller values of $|M_1|$.

Let us compute the value of η_B in the generic case. From eqs. (2.16), (3.2) and (3.6) we get

$$\tilde{m}_1 \approx \frac{|m_{ee}|^2 + |m_{e\mu}|^2 + |m_{e\tau}|^2}{|m_{ee}|} = \frac{|m_2|^2 s_{12}^2 + |m_1|^2 c_{12}^2}{|m_2 s_{12}^2 + m_1 c_{12}^2|} + \mathcal{O}(s_{13})\,. \tag{3.18}$$

In the case of the hierarchical spectra of light neutrinos, this gives

$$\begin{aligned} &\tilde{m}_1 \approx |m_2| \approx \sqrt{\Delta m^2_{\rm sol}}\,, && \kappa_1(\tilde{m}_1) \approx 0.02 && (\mathrm{NH})\,,\\ &\tilde{m}_1 \approx \frac{|m_2|}{|s_{12}^2 + e^{-2i\rho}c_{12}^2|} \approx \frac{\sqrt{\Delta m^2_{\rm atm}}}{\cos 2\theta_{12} \div 1}\,, && \kappa_1(\tilde{m}_1) \approx 0.001 \div 0.003 && (\mathrm{IH})\,, \end{aligned} \tag{3.19}$$

where κ_1 has been estimated using eq. (2.17). For $|M_1| \ll |M_{2,3}|$, the lepton asymmetry ϵ_1, given by eq. (2.10), can be written as

$$\epsilon_1 \approx -\frac{3}{16\pi}\left[\frac{|M_1|}{|M_2|}\frac{\mathrm{Im}(h^\dagger h)^2_{12}}{(h^\dagger h)_{11}} + \frac{|M_1|}{|M_3|}\frac{\mathrm{Im}(h^\dagger h)^2_{13}}{(h^\dagger h)_{11}}\right]. \tag{3.20}$$

From eqs. (2.11) and (3.6) we get

$$v^2(h^\dagger h)_{11} \approx m_u^2 I_{11}\,, \qquad v^2(h^\dagger h)_{12} \approx m_u m_c I_{12}\,, \qquad v^2(h^\dagger h)_{13} \approx m_u m_t I_{13}\,,$$

where $I_{ij} = I_{ij}(m_{\alpha\beta})$ are order 1 coefficients. Using these relations and expressions (3.2), (3.3) and (3.5) for M_i in eq. (3.20) we find

$$\epsilon_1 = \frac{3}{16\pi}\frac{m_u^2}{v^2}\cdot I(m_{\alpha\beta})\,, \qquad I(m_{\alpha\beta}) \sim 1\,.$$

Then the produced baryon-to-photon ratio is given, up to a factor of order one, by

$$\eta_B \simeq 0.01 \cdot \epsilon_1 \cdot \kappa_1(\tilde{m}_1) \simeq 4 \cdot 10^{-16} \cdot \left(\frac{m_u}{1\,\mathrm{MeV}}\right)^2\left(\frac{\kappa_1(\tilde{m}_1)}{0.02}\right).$$

To reproduce the observed value of η_B, one would need $m_u \sim 1\,\mathrm{GeV}$. Thus, a successful leptogenesis requires $m_u \sim m_c$, which contradicts our assumption of a strong hierarchy between the eigenvalues of m_D and goes contrary to the simple GUT expectations.

Our conclusions concerning the mass spectrum of RH neutrinos and the generated baryon asymmetry in the generic case are in accord with those reached in the previous studies [11]–[14].

4. Special cases and level crossing

The results of the previous section were essentially based on two assumptions: (1) $m_{ee} \neq 0$ and is of the order of other elements of m, so that the evaluations (3.2) and (3.3) of $M_{1,2}$ are valid, and (2) $m_{ee}m_{\mu\mu} - m_{e\mu}^2 \neq 0$, so that the evaluations (3.3) and (3.5) of $M_{2,3}$ hold. Let us analyze the situations when one of these conditions or both of them are not satisfied.

Special case I:

$$m_{ee} \to 0$$

or, equivalently, $W_{11} \to 0$. Formally, eqs. (3.2) and (3.3) imply that $|M_1| \to \infty$ and $|M_2| \to 0$ when $m_{ee} \to 0$. At some point (the "level crossing" point) they will become equal to each other. The approximate formulas in eqs. (3.2) and (3.3) do not work when m_{ee} becomes very small. In exact calculations one gets a significant decrease of the level splitting and, therefore, strong mass degeneracy. This behavior can be seen in figure 1, where we show the dependence of the RH neutrino masses and of $|m_{ee}|$ on the lightest mass $|m_1|$, for different values of the Majorana phases of the light neutrinos ρ and σ. Small value of m_{ee} appears as a result of a cancellation of different contributions, which can be realized only for $\rho = \pi/2$ (figure 1, panels b and d). At the crossing points the mixing between the levels becomes maximal.[5]

Special case II:

$$d_{12} \equiv m_{ee}m_{\mu\mu} - m_{e\mu}^2 \to 0\,,$$

or, equivalently, $(W_{11}W_{22} - W_{12}^2) \to 0$. In this limit, according to eqs. (3.3) and (3.5), M_2 increases and M_3 decreases, so that a crossing occurs between the N_2 and N_3 levels. At the crossing point the mixing becomes maximal. In figure 1 we show the dependence of $|d_{12}|$ on $|m_1|$. The crossing points coincide with zeros of the (12)-subdeterminant. As we will see in section 6, $|d_{12}|$ is a non-monotonous function of $|m_1|$, so that, depending on the phases ρ and σ, there can be zero (figure 1a), one (figure 1b) or two (fig. 1d) crossings of this type. For $\rho = 0$, $\sigma = \pi/2$ and quasi-degenerate spectrum of light neutrinos (right-hand part of figure 1c), $|d_{12}|$ is much smaller than the squares of the light neutrino masses. This leads to a quasi-degeneracy of N_2 and N_3 without level crossing.

Special case III:

$$m_{ee} \to 0 \qquad \text{and} \qquad d_{12} \to 0\,.$$

This is equivalent to the requirement that the elements m_{ee} and $m_{e\mu}$ be both very small. In this case all three RH neutrino masses are of the same order. The $1-2$ and $2-3$ crossing regions merge.

In figure 2 we show the dependence of $|M_i|$, $|m_{ee}|$ and $|d_{12}|$ on $|m_1|$ for non-zero s_{13} and different values of the Dirac phase δ. Comparing figure 1b and figure 2, which correspond to the same Majorana phases, we find that the effect of s_{13} for zero δ (figure 2a) is reduced to a small shift of the crossing points. A different choice of the phase δ has more substantial effect: it can remove all crossings (figure 2b), remove only the $2-3$ crossing (figure 2c) or

[5]The level crossing of RH neutrinos has been previously discussed in [12].

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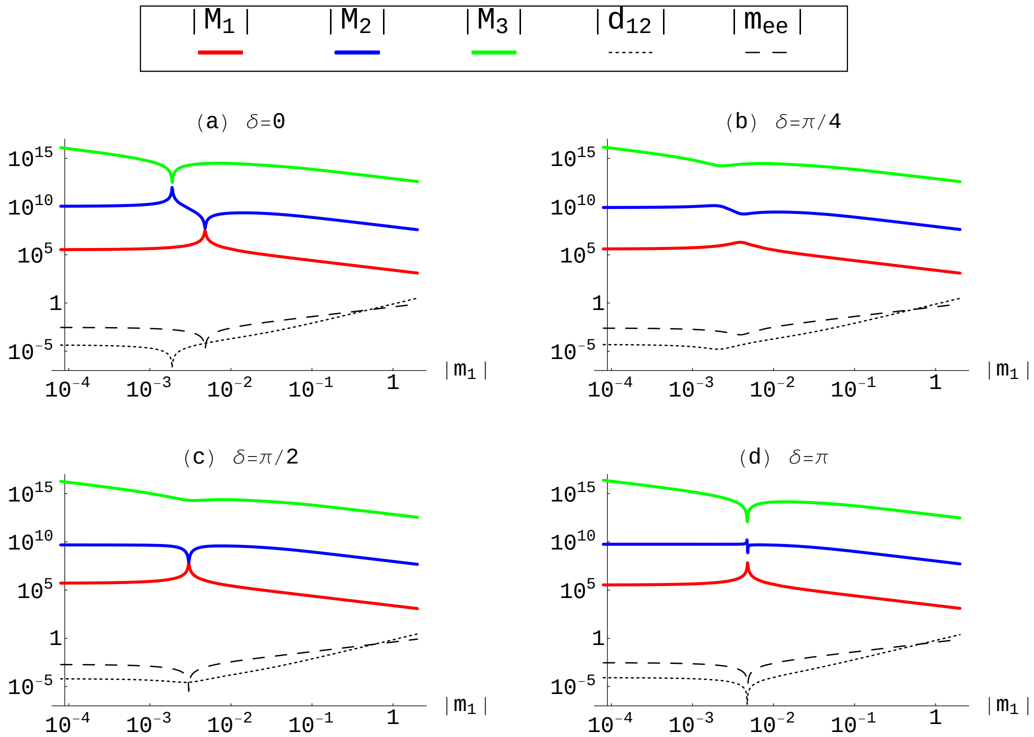

Figure 2: Same as in figure 1, but for $\rho = \pi/2$, $\sigma = 0$, $s_{13} = 0.1$ and different values of the Dirac-type CP-violating phase δ.

change the relative positions of the crossing points leading to quasi-degeneracy of all three RH neutrinos (figure 2*d*).

As one can see in figures 1 and 2, the generic case with a strong hierarchy and small mass of the lightest RH neutrino is realized practically in the whole parameter space, excluding the regions of the crossings. In general, with the increase of the overall scale of the light neutrino mass $|m_1|$, the masses of the RH neutrinos decrease.

In the following sections we will consider the special cases in detail.

5. Special case I: small m_{ee}

Consider the case

$$|m_{ee}| \ll \frac{m_u}{m_c}|m_{e\mu}| \tag{5.1}$$

which corresponds to $|W_{11}| \ll |W_{12}|$ (see eq. (2.24)). In this case the (12)-block of W is dominated by the off-diagonal entries and, to a good approximation, the RH neutrino masses are

$$M_1 \approx -M_2 \approx \frac{1}{W_{12}} \approx -\frac{m_u m_c}{m_{e\mu}}\,, \qquad M_3 \approx \frac{m_t^2 m_{e\mu}^2}{m_1 m_2 m_3}\,. \tag{5.2}$$

Notice that $|M_1|$ is increased by a factor $\sim m_c/m_u$ with respect to the generic case (eq. (3.2)). Moreover, the RH (12)-mixing is nearly maximal while the other mixing angles are very small:

$$U_R \approx \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & \left(\dfrac{m_{\mu\mu}m_{e\tau} - m_{e\mu}m_{\mu\tau}}{m_{e\mu}^2}\right)^* \dfrac{m_u}{m_t} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & -\left(\dfrac{m_{e\tau}}{m_{e\mu}}\right)^* \dfrac{m_c}{m_t} \\ -\dfrac{m_{e\tau}}{\sqrt{2}m_{e\mu}}\dfrac{m_c}{m_t} & \dfrac{m_{e\tau}}{\sqrt{2}m_{e\mu}}\dfrac{m_c}{m_t} & 1 \end{pmatrix} \cdot K \,. \tag{5.3}$$

The matrix of phases K is given in eq. (3.7). Thus, the RH neutrinos N_1 and N_2 are quasi-degenerate, have nearly opposite CP parities and almost maximal mixing ($1-2$ level crossing). The third RH neutrino N_3 is much heavier and weakly mixed with the first two.

Notice that, for $U_L = \mathbb{1}$, m_{ee} is the effective mass directly measurable in the neutrinoless 2β decay experiments. In our parameterization, it is given by

$$m_{ee} = c_{13}^2(m_1 c_{12}^2 + m_2 s_{12}^2) + s_{13}^2 e^{2i\delta} m_3 \,,$$

so that $m_{ee} \approx 0$ implies

$$\tan^2\theta_{13} \approx -\frac{m_1 c_{12}^2 + m_2 s_{12}^2}{e^{2i\delta} m_3} \,. \tag{5.4}$$

For $s_{13} = 0$ the level crossing ($m_{ee} \to 0$) occurs for

$$|m_1| \approx \frac{\tan^2\theta_{12}\sqrt{\Delta m^2_{\rm sol}}}{\sqrt{1-\tan^4\theta_{12}}} \approx (3-4)\cdot 10^{-3}\,\mathrm{eV} \tag{5.5}$$

(see figure 1, panels b and d). For substantial deviations of the $1-2$ mixing from the maximal one ($\tan^2\theta_{12} < 1$), eq. (5.5) can hold only in the case of the normal mass hierarchy. For the inverted hierarchy or quasi-degenerate spectrum one has $|m_1| \gtrsim \sqrt{\Delta m^2_{\rm atm}} \approx 0.05\,\mathrm{eV}$, so that eq. (5.5) is not satisfied. Non-zero s_{13} shifts the position of the level crossing. Taking into account the present upper bound on s_{13}, we find that relation (5.4) can be satisfied for $|m_1| \lesssim 0.02\,\mathrm{eV}$. Moreover, the crossing takes place only for specific values of the phase δ (see figure 2). If a stronger upper bound on θ_{13} is established, eq. (5.4) will provide a more stringent upper bound on $|m_1|$ and also a lower bound on $|m_1|$.

Notice that inequality (5.1) implies

$$|m_{ee}| < 10^{-5}\,\mathrm{eV}\left(\frac{400 m_u}{m_c}\right),$$

where we have taken $|m_{e\mu}|^2 \lesssim \Delta m^2_{\rm sol}$ (the normal hierarchy case). If a positive signal is found in neutrinoless 2β-decay experiments with the near future sensitivity ($|m_{ee}| \gtrsim 0.01\,\mathrm{eV}$ [34]), this special case will be excluded for $U_L = \mathbb{1}$.

Let us consider the effect of possible left-handed Dirac rotations assuming $U_L \sim U_{CKM}$. Taking for simplicity only a 1-2 rotation with $\theta_L \sim \theta_c = 0.22$, we find from eq. (2.23)

$$\hat{m}_{ee} = \cos^2\theta_L m_{ee} + 2\sin\theta_L\cos\theta_L m_{e\mu} + \sin^2\theta_L m_{\mu\mu} \,.$$

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Then the $1-2$ crossing condition, $\hat{m}_{ee} \to 0$, leads to the following restriction on the possible values of m_{ee}:

$$m_{ee} \approx -2 \tan\theta_L m_{e\mu} - \tan^2\theta_L m_{\mu\mu}\,, \tag{5.6}$$

which can be considered as the level crossing condition in the flavor basis. For the case of the normal mass hierarchy $|m_{e\mu}| \sim 0.5\sqrt{\Delta m^2_{\rm sol}}$ and $|m_{\mu\mu}| \sim 0.5\sqrt{\Delta m^2_{\rm atm}}$ and from (5.6) we find $|m_{ee}| \leq (3-4)\cdot 10^{-3}\,{\rm eV}$.

The level crossing condition (5.6) can be satisfied also for the inverted mass hierarchy of light neutrinos as well as for the degenerate spectrum, if ν_1 and ν_2 have opposite CP parities. In the case of the inverted hierarchy one has $|m_{e\mu}| \sim 0.5\sqrt{\Delta m^2_{\rm atm}} > |m_{\mu\mu}|$ [23], so that eq. (5.6) implies $|m_{ee}| \leq (1-2)\cdot 10^{-2}\,{\rm eV}$. For the degenerate spectrum (taking into account the cosmological bound (2.4)) one finds [22] $|m_{e\mu}| \sim |m_{\mu\mu}| \sim 0.2-0.4\,{\rm eV}$, and consequently $|m_{ee}| \leq 0.2\,{\rm eV}$.

In the limit $\theta_{13} = 0$, we find from eqs. (5.2) and (5.4)

$$|M_{1,2}| \approx \frac{2\sqrt{\cos 2\theta_{12}}}{c_{23}\sin 2\theta_{12}} \frac{m_u m_c}{\sqrt{\Delta m^2_{\rm sol}}} \approx 9\cdot 10^7\,{\rm GeV}\left(\frac{m_u}{1\,{\rm MeV}}\right)\left(\frac{m_c}{400\,{\rm MeV}}\right)\,, \tag{5.7}$$

$$|M_3| \approx \frac{c_{23}^2 m_t^2}{\sqrt{\Delta m^2_{\rm atm}}} \approx 10^{14}\,{\rm GeV}\left(\frac{m_t}{100\,{\rm GeV}}\right)^2 \tag{5.8}$$

(see panels b and d of figure 1).

Let us now consider the predictions for leptogenesis. Since N_1 and N_2 are quasi-degenerate and almost maximally mixed, one expects nearly equal contributions to η_B from their decays. Using eqs. (5.2) and (5.3), we find from (2.16)

$$\tilde{m}_1 \approx \tilde{m}_2 \approx \frac{m_c}{m_u}\frac{|m_{e\mu}|^2 + |m_{e\tau}|^2}{2|m_{e\mu}|} = \frac{m_c}{m_u}\left(\frac{s_{12}c_{12}}{2c_{23}}|m_2 - m_1| + \mathcal{O}(s_{13})\right).$$

Therefore, the maximal RH mixing in the (12)-sector leads to an increase of $\tilde{m}_1$ by a factor $\sim m_c/m_u$ with respect to the generic case (eq. (3.18)) and, as a consequence, to a strong enhancement of the washout effects. Taking into account eq. (5.4), we obtain for $s_{13} \approx 0$

$$\tilde{m}_1 \approx \frac{m_c}{m_u}\frac{\sin 2\theta_{12}\sqrt{\Delta m^2_{\rm sol}}}{4c_{23}\sqrt{\cos 2\theta_{12}}} \approx 1.5\,{\rm eV}\left(\frac{m_c}{400 m_u}\right), \tag{5.9}$$

and then, according to eq. (2.17),

$$\kappa_1(1.5\,{\rm eV}) \approx 6\cdot 10^{-5}\,. \tag{5.10}$$

The washout effects for lepton asymmetries produced in the decays of N_1 and N_2 are nearly the same: $\kappa_1(\tilde{m}_1) \approx \kappa_2(\tilde{m}_2)$.

Substituting the mixing parameters given by eq. (5.3) into eq. (2.11), we find the relevant entries of $(h^\dagger h)$:

$$(h^\dagger h)_{12} \approx (h^\dagger h)^*_{21} \approx -\frac{1}{2}\frac{m_c^2}{v^2}\left(1 + \left|\frac{m_{e\tau}}{m_{e\mu}}\right|^2\right) e^{i(\phi_1-\phi_2)/2}\,,$$

$$(h^\dagger h)_{11} \approx (h^\dagger h)_{22} \approx \frac{1}{2}\frac{m_c^2}{v^2}\left(1+\left|\frac{m_{e\tau}}{m_{e\mu}}\right|^2\right),$$
$$(h^\dagger h)_{13} \approx e^{i(\phi_1-\phi_2)/2}(h^\dagger h)_{23} \approx -\frac{1}{\sqrt{2}}\frac{m_c m_t}{v^2}\left(\frac{m_{e\tau}}{m_{e\mu}}\right)^* e^{i(\phi_1-\phi_3)/2}\,. \tag{5.11}$$

Then the contribution to $\epsilon_{1,2}$ coming from the diagrams with the heaviest RH neutrino N_3 in the loop (terms with $k=3$ in eq. (2.10)) can be estimated as follows:

$$\epsilon_{1,2}^{(N_3)} \sim \pm\frac{3}{16\pi}\frac{m_u m_c}{v^2}\sqrt{\frac{\Delta m^2_{\rm atm}}{\Delta m^2_{\rm sol}}} \approx \pm 5\cdot 10^{-9}\,.$$

These asymmetries are tiny compared to the values required for a successful leptogenesis and, moreover, have opposite signs for ϵ_1 and ϵ_2. Therefore the dominant contribution should come from the diagrams with N_2 (N_1) in the loop for the decay of N_1 (N_2). The corresponding asymmetries $\epsilon_{1,2}$ can be written as

$$\epsilon_1 \approx \epsilon_2 \approx \frac{1}{16\pi}\frac{|M_1|}{|M_1|-|M_2|}\frac{{\rm Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \approx \frac{1}{32\pi}\frac{m_c^2}{v^2}\left(1+\left|\frac{m_{e\tau}}{m_{e\mu}}\right|^2\right)\xi\,, \tag{5.12}$$

where

$$\xi = \frac{|M_1|}{|M_1|-|M_2|}\sin(\phi_1-\phi_2)\,. \tag{5.13}$$

The enhancement due to the quasi-degeneracy of N_1 and N_2 competes with the suppression due to their almost opposite CP parities. Indeed, in the limit of exactly vanishing W_{11} and W_{22}, one has $\sin(\phi_1-\phi_2)=\sin\pi=0$: in this case the complex phases can be removed by the transformation (2.14). Taking into account terms of order W_{11} and W_{22}, we find

$$\xi \approx \frac{4k\tan\Delta}{(1+k)^2+(1-k)^2\tan^2\Delta}\,, \tag{5.14}$$

where

$$k \equiv \frac{|W_{22}|}{|W_{11}|} = \frac{m_u^2|m_{\mu\mu}|}{m_c^2|m_{ee}|}\,, \qquad \Delta \equiv \frac{1}{2}\arg\frac{W_{12}^2}{W_{11}W_{22}} = \frac{1}{2}\arg\frac{m_{e\mu}^2}{m_{ee}m_{\mu\mu}}\,. \tag{5.15}$$

Notice that the phase Δ is invariant under the transformation (2.14). For $|1-k| \ll 1/\tan\Delta$, eq. (5.14) gives

$$\xi \approx \tan\Delta\,,$$

and for $\Delta \simeq \pi/2$ a significant enhancement of the asymmetries $\epsilon_{1,2}$ can be achieved. The enhancement factor depends on the degree of near-equality of $|W_{22}|$ and $|W_{11}|$.

For $k\to 1$, the level splitting can be written as

$$\frac{|M_2|-|M_1|}{|M_1|} \approx 2\frac{|W_{22}|}{|W_{12}|}\cos\Delta = 2\frac{|m_{\mu\mu}|}{|m_{e\mu}|}\frac{m_u}{m_c}\cos\Delta\,, \tag{5.16}$$

so that for $\Delta\approx\pi/2$ the splitting is substantially reduced. As we discussed in section 2.3, the enhancement due to the degeneracy is restricted by the condition

$$\frac{|M_2|-|M_1|}{|M_1|} \gtrsim \frac{\Gamma_1}{|M_1|}\,, \tag{5.17}$$

where in the case under the consideration

$$\frac{\Gamma_1}{|M_1|} \approx \frac{1}{8\pi}\frac{m_c^2}{2v^2}\left(1+\left|\frac{m_{e\tau}}{m_{e\mu}}\right|^2\right). \tag{5.18}$$

Estimating $|m_{e\mu}| \approx |m_{e\tau}| \approx |m_{\mu\mu}|\sqrt{\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}}$, from eqs. (5.16), (5.17) and (5.18) we find the maximal possible enhancement:

$$\xi^{\max} \approx \tan\Delta \approx \frac{1}{\cos\Delta} \approx \frac{16\pi v^2 m_u}{m_c^3}\sqrt{\frac{\Delta m^2_{\rm atm}}{\Delta m^2_{\rm sol}}} \approx 1.4\cdot 10^5\,. \tag{5.19}$$

In the numerical estimate we have taken the values in eq. (2.8). Using eqs. (5.16), (5.18) and (5.12), we can write the maximal asymmetry as

$$\epsilon^{\max} = \frac{1}{2}\frac{\Gamma_1}{|M_1|}\xi^{\max} \approx \frac{|W_{22}|}{|W_{12}|} \approx \frac{m_u}{m_c}\sqrt{\frac{\Delta m^2_{\rm atm}}{\Delta m^2_{\rm sol}}}\,,$$

which shows that $\epsilon^{\max} \sim 10^{-2}$ is reachable in this scenario.

Combining eqs. (2.17), (5.12) and (5.19), we find

$$\eta_B \approx 0.01\cdot 2\epsilon_1\kappa_1 \approx 1.9\cdot 10^{-8}\left(\frac{400 m_u}{m_c}\right)^2\left[1+0.14\log\left(\frac{m_c}{400 m_u}\right)\right]^{-0.6}\left[\frac{\xi}{\xi^{\max}(m_u,m_c)}\right].$$

Therefore the value (2.9) of η_B can be obtained for $m_u/m_c \gtrsim 2\cdot 10^{-3}$. For $m_c = 400 m_u$, the observed baryon asymmetry is reproduced for $\xi \approx \tan\Delta \approx 5\cdot 10^3$, which corresponds to the relative splitting (see eq. (5.16))

$$\frac{|M_2|-|M_1|}{|M_1|} \approx 6\cdot 10^{-6}\,.$$

Thus, in spite of strong washout effects, a sufficiently large baryon asymmetry can be generated in this case, due to the enhancement related to the strong degeneracy of the RH neutrinos. For this to occur, not only the level crossing condition ($m_{ee} \to 0$) has to be satisfied, but also a special phase condition leading to $\Delta \approx \pi/2$ should be fulfilled. This value of Δ is consistent with the low energy neutrino data. We have checked the analytic results presented in this section by precise numerical calculations.

6. Special case II: small 12-subdeterminant of m

Let us consider the case in which the (11)-element of the matrix W in eq. (2.24) is still the dominant one (as in the generic case), but the (12)-subdeterminant of W is very small. Then $(M_R)_{33}$, which is proportional to this subdeterminant, is suppressed. The condition $(M_R)_{33} \ll (M_R)_{23}$ can be written as

$$|d_{12}| \equiv |m_{ee}m_{\mu\mu} - m^2_{e\mu}| \ll \frac{m_c}{m_t}|m_{e\tau}m_{e\mu} - m_{ee}m_{\mu\tau}|\,. \tag{6.1}$$

In this case M_1 is still given by eq. (3.2), but M_2 cannot be found from the determinant of the (12)-block of W as in eq. (3.3). One has to consider, instead, the (23)-block of M_R, which is dominated by its off-diagonal entry. This yields

$$M_2 \approx -M_3 \approx (M_R)_{23} = \frac{m_c m_t}{m_1 m_2 m_3}(m_{ee} m_{\mu\tau} - m_{e\tau} m_{e\mu})\,. \tag{6.2}$$

The mixing matrix of RH neutrinos equals

$$U_R \approx \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}}\left(\frac{m_{e\mu}}{m_{ee}}\right)^* \frac{m_u}{m_c} & -\frac{1}{\sqrt{2}}\left(\frac{m_{e\mu}}{m_{ee}}\right)^* \frac{m_u}{m_c} \\ \left(\frac{m_{e\mu}}{m_{ee}}\right)\frac{m_u}{m_c} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \left(\frac{m_{e\tau}}{m_{ee}}\right)\frac{m_u}{m_t} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot K\,, \tag{6.3}$$

where K is given in eq. (3.7). From eq. (6.2) it follows that the phases of M_2 and M_3 differ by $\approx \pi$. Therefore in this special case the lightest RH neutrino is weakly mixed with N_2 and N_3, which are much heavier, quasi-degenerate, almost maximally mixed and have nearly opposite CP parities.

Let us consider condition (6.1). In terms of low-energy neutrino parameters, we obtain from eq. (2.1)

$$d_{12} = c_{23}^2 m_1 m_2 + s_{23}^2 m_3 (c_{12}^2 m_1 + s_{12}^2 m_2) + \mathcal{O}(s_{13})\,.$$

Neglecting $\mathcal{O}(s_{13})$ corrections, we find that d_{12} vanishes for

$$m_3 = -\frac{\cot^2\theta_{23} m_1 m_2}{c_{12}^2 m_1 + s_{12}^2 m_2}\,. \tag{6.4}$$

Since $\cot^2\theta_{23} \approx 1$, this relation cannot be satisfied for $|m_3| < |m_{1,2}|$. Therefore this special case is not realized for the inverted ordering of the light neutrino masses, unless U_L deviates significantly from $\mathbb{1}$. For the normal mass ordering, condition (6.4) requires $|m_1| \gtrsim 2 \cdot 10^{-3}$ eV (see figure 1, panels b and d). For the quasi-degenerate mass spectrum, eq. (6.4) can be satisfied if the CP parity of ν_3 is opposite to the CP parities of ν_1 and ν_2, up to deviations of θ_{23} from $\pi/4$ (see the right-hand side of figure 1*c*). Notice that

$$\begin{aligned} m_{e\tau} m_{e\mu} - m_{ee} m_{\mu\tau} &= s_{23} c_{23} \left[m_1 m_2 - m_3 \left(c_{12}^2 m_1 + s_{12}^2 m_2 \right) \right] + \mathcal{O}(s_{13}) \\ &= \cot\theta_{23} m_1 m_2 + \mathcal{O}(s_{13})\,, \end{aligned} \tag{6.5}$$

where we have used the condition (6.4) of zero (12)-subdeterminant. Therefore eq. (6.2) simplifies to

$$M_2 \approx -M_3 \approx -\frac{m_c m_t}{m_3}\,. \tag{6.6}$$

Numerically, one finds (see figure 1, panels *b*, *c*, *d*)

$$|M_2| \approx |M_3| \approx \frac{m_c m_t}{(0.05 - 0.7)\,\text{eV}} \approx (0.6 - 8) \cdot 10^{11}\,\text{GeV} \left(\frac{m_c}{400\,\text{MeV}}\right)\left(\frac{m_t}{100\,\text{GeV}}\right), \tag{6.7}$$

while $|M_1|$ is still given by eqs. (3.11) and (3.17) for the normal hierarchy and quasi-degenerate case, respectively.

For this special case, the predictions for the lepton asymmetry are analogous to those in the generic case. The production of the asymmetry is dominated by the decays of the lightest RH neutrino. Due to the larger RH mixing, the asymmetry ϵ_1 gets an enhancement factor $\sim (m_t/m_c)$ with respect to the generic case, but the leading terms in $\mathrm{Im}(h^\dagger h)^2_{12}$ and $\mathrm{Im}(h^\dagger h)^2_{13}$ cancel because of the nearly opposite CP parities of N_2 and N_3. Indeed, the sum of the two terms is proportional to $\sin(\phi_2 - \phi_3) \approx 0$, where ϕ_i are defined in eq. (3.7). Thus, the produced lepton asymmetry is insufficient for a successful baryogenesis through leptogenesis. This is in agreement with the fact that in this special case the value of $|M_1|$ is still below the absolute lower bound (2.18).

7. Special case III: small m_{ee} and $m_{e\mu}$

Consider now the case when

$$|m_{ee}| \ll \frac{m_u}{m_t}|m_{e\tau}|\,, \qquad |m_{e\mu}| \ll \frac{m_c}{m_t}|m_{e\tau}|\,, \qquad \frac{m_u}{m_c}|m_{\mu\mu}|\,, \tag{7.1}$$

so that the (13)- and (22)-elements of $W \equiv M_R^{-1}$ are the dominant ones (see eq. (2.24)). In this case, two RH neutrinos form a quasi-degenerate pair with almost maximal mixing, opposite CP-parities and masses

$$\pm M_d \approx \pm W_{13}^{-1} \approx \mp \frac{m_u m_t}{m_{e\tau}}\,. \tag{7.2}$$

The third neutrino has small mixing with the other two (of order m_u/m_c or m_c/m_t) and a mass

$$M_s \approx W_{22}^{-1} \approx -\frac{m_c^2}{m_{\mu\mu}}\,. \tag{7.3}$$

Since $m_u m_t \sim m_c^2$, all the three masses are of the same order.[6]

Let us consider conditions (7.1). We have shown in section 5 that, assuming $U_L \approx \mathbb{1}$, m_{ee} can be very small only in the case of the normal hierarchy, when eq. (5.4) can be satisfied. At the same time, for certain values of m_1 and s_{13} and of the phases, the value of $m_{e\mu}$ can also be very small. For this to occur, the low-energy parameters should satisfy (see figure 2d)

$$|m_1| \approx |m_2| \tan^2\theta_{12} \approx 0.0035\,\mathrm{eV}\,, \qquad s_{13} \approx \left|\frac{m_2}{m_3}\right| \tan\theta_{12} \approx 0.11\,,$$

where we used $|m_2| \approx \sqrt{\Delta m^2_{\mathrm{sol}}}$ and $|m_3| \approx \sqrt{\Delta m^2_{\mathrm{atm}}}$. The other matrix elements of m are also approximately determined by the conditions $m_{ee} \approx 0$, $m_{e\mu} \approx 0$:

$$|m_{e\tau}| \approx \sqrt{2}\tan\theta_{12}|m_2| \approx 0.008\,\mathrm{eV}\,, \qquad |m_{\mu\mu}| \approx |m_{\mu\tau}| \approx |m_{\tau\tau}| \approx \frac{|m_3|}{2} \approx 0.025\,\mathrm{eV}\,.$$

[6] Seesaw mass matrices which correspond to the degeneracy of all three RH neutrinos have been recently considered in [35, 36].

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For the masses of the RH neutrinos one then finds

$$|M_d| \simeq 1.2 \times 10^{10}\,\text{GeV} \left(\frac{m_u}{1\,\text{MeV}}\right)\left(\frac{m_t}{100\,\text{GeV}}\right),$$
$$|M_s| \simeq 6.4 \cdot 10^{9}\,\text{GeV} \left(\frac{m_c}{400\,\text{MeV}}\right)^2 . \tag{7.4}$$

According to eqs. (7.2) and (7.3), the mass spectrum of RH neutrinos is characterized by the ratio

$$r \equiv \frac{|M_d|}{|M_s|} = \frac{m_u m_t}{m_c^2}\left|\frac{m_{\mu\mu}}{m_{e\tau}}\right| = 1.9\left(\frac{m_u}{1\,\text{MeV}}\right)\left(\frac{m_t}{100\,\text{GeV}}\right)\left(\frac{400\,\text{MeV}}{m_c}\right)^2\left|\frac{m_{\mu\mu}}{3m_{e\tau}}\right| . \tag{7.5}$$

We shall distinguish two subcases, depending on whether the quasi-degenerate pair is lighter or heavier than the singlet state:

(*a*) $r < 1 \;\Rightarrow\; |M_1| \approx |M_2| \lesssim |M_3|$.

(*b*) $r > 1 \;\Rightarrow\; |M_1| \lesssim |M_2| \approx |M_3|$.

For certain values of the parameters, the splitting between the quasi-degenerate neutrinos can become larger than the difference between the masses of one of them and of the third neutrino. This case can be considered as the limit in which (a) and (b) merge. Notice that, in this limit, the structure of U_R is very unstable, and all three RH mixing angles can be large.

Let us consider now the predictions for leptogenesis. To compute the produced lepton asymmetry one has to take into account the interplay among all three quasi-degenerate RH neutrinos. The effects related to mass degeneracy and large RH mixing angles, discussed in section 5, are present also here. Notice that the maximal RH mixing is now related with the Dirac masses m_u and m_t rather than with m_u and m_c, as in section 5. Let us discuss the two subcases defined above.

(*a*) $r < 1$, light quasi-degenerate pair.

Up to $\mathcal{O}(\lambda^2)$ terms, the RH mixing matrix is given by

$$U_R \approx \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \delta_1^* \\ \frac{1}{\sqrt{2}}(\delta_2^* - \delta_1) & -\frac{1}{\sqrt{2}}(\delta_2^* + \delta_1) & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & \delta_2 \end{pmatrix} \cdot K' . \tag{7.6}$$

Here

$$\delta_1 \equiv \frac{m_u}{m_c}\,\frac{m_{\mu\tau}}{m_{e\tau}}\,\frac{1}{r^2-1}\,, \qquad \delta_2 \equiv \frac{m_u}{m_c}\left(\frac{m_{\mu\tau}}{m_{e\tau}}\right)^*\frac{r}{r^2-1}\,, \tag{7.7}$$

and $K' = \text{diag}(e^{-i\phi_1'/2},\ e^{-i\phi_2'/2},\ e^{-i\phi_3'/2})$, where

$$\phi_1' - \phi_2' \approx \pi\,, \qquad \phi_3' - \phi_2' \approx 2\arg\left(\frac{m_{e\tau}}{m_{\mu\tau}}\right) . \tag{7.8}$$

Notice that in the parameterization (7.6) the phases ϕ_i' are not the arguments of M_i.

From eqs. (7.2), (7.6) and (2.16) we find the effective mass parameter

$$\tilde{m}_1 \approx \tilde{m}_2 \approx \frac{m_t}{2m_u}|m_{e\tau}| \approx 500\,\mathrm{eV}\,,$$

which leads, according to eq. (2.17), to a very small washout factor

$$\kappa_{1,2}(500\,\mathrm{eV}) \approx 10^{-7}\,. \tag{7.9}$$

To survive such strong a washout, lepton asymmetries ϵ_i of order unity are required.

The contribution to $\epsilon_{1,2}$ of diagrams with N_3 in the loop can be estimated as

$$\epsilon_{1,2}^{(N_3)} \sim \pm\frac{3}{16\pi}\frac{m_c^2}{v^2} \approx \pm 3\cdot 10^{-7}\,,$$

where we assumed $|M_3|/|M_{1,2}| \gtrsim 1.5$, so that the effects of the three-neutrino degeneracy can be disregarded. Let us estimate the contribution to $\epsilon_{1,2}$ of diagrams with $N_{2,1}$ in the loop. The maximal asymmetry is obtained when

$$\frac{|M_2|-|M_1|}{|M_1|} \approx \frac{\Gamma_{1,2}}{|M_1|} \approx \frac{m_t^2}{16\pi v^2}\,. \tag{7.10}$$

In this case the function f in eq. (2.10) should be replaced by $|M_1|/(2\Gamma_1)$ [29], and one finds

$$\epsilon_1 \approx \epsilon_2 \approx \frac{1}{16\pi}\frac{|M_1|}{\Gamma_1}\frac{\mathrm{Im}(h^\dagger h)_{12}^2}{(h^\dagger h)_{11}} \approx \frac{1}{2}\sin(\phi_1' - \phi_2')\,. \tag{7.11}$$

As in the special case I, the factor $\sin(\phi_1' - \phi_2')$ is suppressed because of the approximately opposite CP parities of N_1 and N_2 (see eq. (7.8)). Computing also $\mathcal{O}(\lambda^2)$ terms in U_R, we find

$$\sin(\phi_1' - \phi_2') \sim \frac{m_u}{m_t} \approx 10^{-5}\,. \tag{7.12}$$

As far as ϵ_3 is concerned, the two contributions proportional to $\mathrm{Im}(h^\dagger h)_{31}^2$ and $\mathrm{Im}(h^\dagger h)_{32}^2$ are of order $m_t^2/(16\pi v^2)$, but have opposite sign because of the opposite CP parities of N_1 and N_2. Moreover, ϵ_3 is washed out efficiently by the strong L-violating interactions of N_1 and N_2. Therefore its contribution to η_B can be neglected and we finally obtain

$$\eta_B \approx 0.01\cdot 2\epsilon_1\kappa_1 \sim 10^{-14}\left(\frac{10^5 m_u}{m_t}\right)\left(\frac{\kappa_1(m_u/m_t)}{10^{-7}}\right).$$

Thus, the leptogenesis is not successful in this special case.

Reducing the ratio m_t/m_u, one gets both smaller washout and enhanced asymmetries $\epsilon_{1,2}$ (see eqs. (7.9), (7.11) and (7.12)). However, to obtain η_B in the correct range, one would have to violate the assumption in eq. (2.7). Even a strong degeneracy between all three RH neutrinos cannot lead to a sufficient increase of the final baryon asymmetry, because the enhancement due to the degeneracy is limited by the large values of $\Gamma_{1,2}$ given in eq. (7.10) (see also the discussion at the end of this section). Moreover, the analytic approximation (2.17) most probably underestimates the washout effects in this case, because of the very large values of $\tilde{m}_1$ and $\tilde{m}_2$ ($\sim$ 500 eV). To the best of our knowledge, no numerical solutions of the relevant Boltzmann equations in this regime are available in the literature, since it is usually assumed that $\tilde{m}_1$ does not exceed the mass of the heaviest left-handed neutrino $|m_3|$. The present special case shows that this is not always true.

(*b*) $r > 1$, heavy quasi-degenerate pair.

In this case ϵ_1 gives the dominant contribution to the final baryon asymmetry. The RH mixing matrix is obtained from that in eq. (7.6) by the cyclic permutation of its columns $3 \to 1$, $1 \to 2$, $2 \to 3$. Using the approximation

$$\delta_2 \approx \frac{m_c}{m_t} \frac{r^2}{r^2 - 1} e^{-i\phi/2} \,,$$

we obtain from eq. (2.11)

$$\begin{aligned} (h^\dagger h)_{11} &\approx \frac{m_c^2}{v^2} \frac{1 - 2r^2 + 2r^4}{1 - 2r^2 + r^4} \,, \\ (h^\dagger h)_{12} &\approx -e^{i(\phi_3' - \phi_2')/2} (h^\dagger h)_{13} \approx \frac{m_t m_c}{\sqrt{2} v^2} \frac{r^2}{1 - r^2} e^{i(\phi + \phi_1' - \phi_2')/2} \,. \end{aligned} \tag{7.13}$$

Then for $\tilde{m}_1$ and κ_1 we find

$$\tilde{m}_1 \gtrsim 2 |m_{\mu\mu}| \approx 0.05 \,\text{eV} \,, \qquad \kappa_1(\tilde{m}_1) \lesssim 3 \cdot 10^{-3} \,.$$

The asymmetry produced in the decays of N_1 can be written as

$$\epsilon_1 \approx -\frac{3 m_t^2}{32 \pi v^2} \frac{r^3}{1 - 2r^2 + 2r^4} \sin\psi \sin(\phi_3' - \phi_2') \approx 3 \cdot 10^{-3} \;\; \sin(\phi_3' - \phi_2') \,,$$

where $\psi \equiv (\phi + \phi_1' - \phi_2'/2 - \phi_3'/2)$. In the last equality we have chosen $r = 2$ and $\sin\psi = 1$, which are the most favorable values for obtaining a large η_B (note that, even though ϵ_1 is maximized at $r \simeq 1$, in the limit $r \to 1$ the parameter $\tilde{m}_1$ becomes very large, which signals a very strong washout of the asymmetry). Also in this case the fact that the CP parities of N_2 and N_3 are almost opposite leads to a strong cancellation:

$$\sin(\phi_3' - \phi_2') \sim \frac{m_u}{m_t} \approx 10^{-5} \,.$$

Thus we obtain

$$\eta_B \approx 0.01 \cdot \epsilon_1 \cdot \kappa_1 \lesssim 5 \cdot 10^{-13} \left(\frac{m_u}{1 \,\text{MeV}} \right) \left(\frac{m_t}{100 \,\text{GeV}} \right) .$$

Increasing $m_u \cdot m_t$ (and also m_c^2 in order to keep r fixed) would increase η_B.

However, it is unlikely that this would lead to a successful leptogenesis. Indeed, since all three RH neutrino masses are of the same order in this special case ($r \sim 1$), the heavier neutrinos N_2 and N_3 are still abundant at the temperature $T \sim |M_1|$ at which the decays of N_1 take place. Therefore the strong washout effects due to the processes involving N_2 and N_3, which are characterized by very large $\tilde{m}_{2,3} \simeq 500 \,\text{eV}$, are expected to efficiently wash out the asymmetry ϵ_1 even though the parameter $\tilde{m}_1$ is relatively small. Therefore we do not expect this special case to lead to a successful baryogenesis through leptogenesis. A more accurate study of the case of three quasi-degenerate RH neutrinos would require solving numerically a coupled set of Boltzmann equations describing the evolution of the number densities of all RH neutrinos and $B - L$. We consider such a study, which is beyond the scope of the present paper, to be very desirable.

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8. Discussion and conclusions

We have analyzed the possibility of explaining both the low energy neutrino data and the observed baryon asymmetry of the Universe in the framework of the seesaw mechanism and studied the requisite structure of the RH neutrino sector. Our analysis was based on the assumptions of hierarchical eigenvalues of the Dirac mass matrix m_D and small Dirac-type left-handed mixing ($U_L \approx \mathbb{1}$).

Let us now abandon the hypothesis $U_L \approx \mathbb{1}$. If the matrix U_L is arbitrary, the direct connection between the low energy data and the structure of M_R is lost. This additional freedom relaxes the phenomenological constraints on RH neutrinos. In fact, now the unique low energy requirement on the seesaw mechanism is to reproduce the light neutrino masses, given by the eigenvalues of $\hat{m}$ (see eq. (2.23)); the correct leptonic mixing matrix U_{PMNS} can always be obtained through the proper choice of U_L. As an example, let us consider the case of non-degenerate RH masses and take the following RH mixing matrix:

$$U_R = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot K \,.$$

The maximal mixing of the two lighter RH neutrinos will maximize the lepton asymmetry. The eigenvalues of the matrix

$$\hat{m} = -m_D^{\rm diag} W m_D^{\rm diag}$$

(see eq. (2.22)) are given, approximately, by $m_c^2/(4|M_2|)$, $m_c^2/(2|M_1|)$, $m_t^2/|M_3|$. Taking $|M_1| \approx 10^{10}\,{\rm GeV}\cdot(m_c/0.4\,{\rm GeV})^2$, $|M_2|$ a few times larger and $|M_3| \approx 2\cdot 10^{14}\,{\rm GeV}\cdot(m_t/100\,{\rm GeV})^2$, one can reproduce the solar and atmospheric mass squared differences. Since $\hat{m}$ is approximately diagonal, the solar and atmospheric mixing angles are generated by U_L, which should have an almost bimaximal form.

It is easy to calculate the washout mass parameter and the asymmetry produced in the decays of N_1:

$$\tilde{m}_1 = \frac{m_c^2}{2|M_1|} \approx \sqrt{\Delta m^2_{\rm sol}}\,, \qquad \epsilon_1 \approx \frac{3m_c^2}{32\pi v^2}\sin(\phi_2-\phi_1)\frac{|M_1|}{|M_2|}\,.$$

Assuming $\phi_2 - \phi_1 \sim \pi/2$ (note that the CP parities of N_1 and N_2 are in general not constrained), we get

$$\eta_B \approx 3\cdot 10^{-11}\frac{|M_1|}{|M_2|}\left(\frac{m_c}{0.4\,{\rm GeV}}\right)^2\,.$$

Thus, for a moderate hierarchy between M_1 and M_2, a value of m_c around a few GeV can lead to a successful leptogenesis. This example shows that, relaxing the hypothesis $U_L \approx \mathbb{1}$, it is easier to realize baryogenesis via leptogenesis. In particular, the degeneracy of the masses of RH neutrinos $|M_i|$ is no longer necessary, but the hierarchy of $|M_i|$ should not be as large as it is in the generic case.

Let us now discuss the renormalization group equation (RGE) evolution of the neutrino mass matrices. The structure of the effective mass matrix m is stable under the Standard

Model (or MSSM) radiative corrections [37]. The corrections to its matrix elements can be written as

$$\Delta m_{\alpha\beta} \approx (\epsilon_\alpha + \epsilon_\beta) m_{\alpha\beta} \,,$$

where ϵ_α ($\lesssim 10^{-2}$) describes the effect of the Yukawa coupling of the charged lepton l_α. Therefore both m_{ee} and $d_{12} \equiv (m_{ee}m_{\mu\mu} - m_{e\mu}^2)$ receive small corrections proportional to themselves: if m_{ee} and/or d_{12} are very small at the electroweak scale, they remain very small also at the seesaw scale (the mass scale of RH neutrinos), and so the level crossing conditions do not change.

Between the GUT and the seesaw scales one has to consider the evolution of the neutrino Yukawa couplings and of Majorana mass matrix of RH neutrinos rather than the evolution of the effective matrix m [38]. We assume that at the GUT scale the Yukawa couplings of neutrinos h are related with those of quarks or charged leptons. The evolution of h with decreasing mass scale will not modify the hierarchy $m_u \ll m_c \ll m_t$, and its effects can be absorbed into a redefinition of our indicative values of $m_{u,c,t}$.

The RGE effects on M_R are due to the neutrino Yukawa couplings; they can, in principle, be important in the cases of strongly degenerate RH neutrinos. Consider the stability of the structure of M_R in the special case that leads to a successful leptogenesis. Recall that in this case the (12)-sector of RH neutrinos is characterized by $|M_{1,2}| \approx 10^8\,\text{GeV}$, $(|M_2| - |M_1|)/|M_1| \lesssim 10^{-5}$ and $\Delta \approx \pi/2$, where Δ is defined in eq. (5.15). The largest correction to the (12)-block of M_R between $M_{\rm GUT}$ and $|M_{1,2}|$ is the correction to the 22-element:

$$\frac{(\Delta M_R)_{22}}{(M_R)_{22}} \sim \frac{m_c^2}{16\pi^2 v^2} \log\left(\frac{M_{\rm GUT}}{10^8\,\text{GeV}}\right) \approx 6\cdot 10^{-7}\left(\frac{m_c}{0.4\,\text{GeV}}\right)^2 .$$

Therefore, the radiative corrections cannot generate a relative splitting between $|M_1|$ and $|M_2|$ exceeding 10^{-5}. Moreover, at one loop level, the phases of $(M_R)_{ij}$ have no RGE evolution and so the relation $\Delta \approx \pi/2$ is not modified.

Let us summarize the main results of our analysis:

1. We have discussed the properties of the seesaw mechanism under the assumption of an approximate quark-lepton symmetry, which implies a similarity between the Dirac neutrino mass matrix m_D and the mass matrices of charged leptons and quarks. This, in turn, implies a strong hierarchy of the eigenvalues of m_D and small left-handed Dirac-type mixing.

2. The presence of two large mixing angles (θ_{12} and θ_{23}) and relatively weak mass hierarchy of light neutrinos lead, in general, to a "quasi-democratic" structure of the mass matrix m in the flavor basis, with values of all its elements within one order of magnitude of each other. A strong hierarchy of the elements appears in special cases only.

3. In the generic case (nearly democratic m), the mass matrix of RH neutrinos has a strong (nearly quadratic in m_D) hierarchy of eigenvalues and small mixing. The lightest RH neutrino has a mass $|M_1| < 10^6\,\text{GeV}$, well below the absolute lower

bound coming from the condition of a successful leptogenesis. As a result, the predicted lepton asymmetry is smaller than 10^{-14}, and the scenario of baryogenesis via leptogenesis does not work.

4. The special cases correspond to the level crossing points, when either two or all three masses of RH neutrinos are nearly equal. We have found two level crossing conditions: (1) $m_{ee} \to 0$ (the $N_1 - N_2$ crossing) and (2) $d_{12} \to 0$ ($N_2 - N_3$ crossing), where m_{ee} and d_{12} should be evaluated in the basis where the Yukawa couplings of neutrinos are diagonal. In the crossing points the mixing of the corresponding neutrino states is maximal and their CP parities are nearly opposite.

5. For $U_L \simeq \mathbb{1}$ the leptogenesis can be successful only in the special case with small element m_{ee}, which corresponds to the $N_1 - N_2$ crossing. It is characterized by $|M_1| \approx |M_2| \approx 10^8\,\mathrm{GeV}$, $|M_3| \approx 10^{14}\,\mathrm{GeV}$ and $(|M_2| - |M_1|)/|M_2| \lesssim 10^{-5}$. N_1 and N_2 are strongly mixed and their mixing with N_3 is very small. The CP-violating phase Δ in eq. (5.15) should be very close to $\pi/2$. Notice that this unique case with a successful leptogenesis is defined very precisely. It has a number of characteristic features which can give important hints for model building.

6. For $U_L = \mathbb{1}$, the successful scenario is realized for the normal mass hierarchy of light neutrinos and predicts a very small effective Majorana mass probed in the neutrinoless 2β-decay: $|m_{ee}| \lesssim 10^{-4}\,\mathrm{eV}$. However, for $U_L \approx U_{CKM}$, this case can be realized also for other mass spectra and $|m_{ee}|$ as large as $\sim 0.1\,\mathrm{eV}$.

7. We find that low-energy neutrino data allow also the other special cases, with $2-3$ crossing or both $1-2$ and $2-3$ crossings of the masses of RH neutrinos. These cases, however, do not lead to a successful baryogenesis through leptogenesis.

The seesaw mechanism can account for both the low-energy neutrino data and a successful thermal leptogenesis, but a very specific structure of the mass matrices is required. Although this structure may look as an extreme fine tuning when viewed from the low-energy (effective theory) side ($m_{ee} \to 0$, $|m_{\mu\mu}| \approx (m_u/m_c)^2|m_{ee}|$), it does not appear unnatural from the point of view of the fundamental physics responsible for the seesaw mechanism: indeed, it just requires an approximate degeneracy and nearly maximal mixing of the two lightest RH neutrinos, which may well be a consequence of some flavor symmetry operating in the RH sector.

Can the unique successful special case that we found be ruled out? Since it requires a suppression of $|m_{ee}|$, it will be excluded in case of a positive signal of $2\beta0\nu$-decay with $|m_{ee}|$ close to the heaviest of the light neutrino masses (which could be measured in direct neutrino mass search experiments). In that case one will be left with the following alternatives:

- the quark-lepton symmetry is strongly violated: there is no strong hierarchy of the eigenvalues of m_D ($m_u/m_c, m_c/m_t \gtrsim 10^{-1}$) and/or the Dirac-type left-handed mixing is large (the corresponding mixing angles are larger than $\theta_c \approx 0.2$);

- type-I seesaw [1] is not the sole source of neutrino mass; the simplest alternative could be type-II seesaw [39] in which there is an additional contribution from an $\mathrm{SU}(2)_L$-triplet Higgs. Another possibility is that the seesaw is not the true mechanism of neutrino mass generation;

- a mechanism other than the decay of thermally produced RH neutrinos contributes to leptogenesis or the baryon asymmetry of the Universe is generated through a different mechanism, which has nothing to do with leptogenesis.

Note added. Let us comment on the possibility of non-thermal production of the heavy RH neutrinos (see, e.g., [40]), that in principle can lead to a successful leptogenesis for values of the parameters M_1 and $\tilde{m}_1$ for which thermal leptogenesis does not work.

In fact, it is interesting that also non-thermal leptogenesis is strongly constrained in our framework. Consider the generic case (section 3). Since M_1 is relatively light ($\lesssim 10^7\,\mathrm{GeV}$), ϵ_1 is very small. Moreover, as $\tilde{m}_1$ is relatively large ($\gtrsim \sqrt{\Delta m^2_{\mathrm{sol}}}$), the washout effects suppress (at least partially) the asymmetry generated in the decays of non-thermally produced RH neutrinos [41]. As a consequence, even in the non-thermal case, the asymmetry generated by N_1 turns out to be insufficient and, to enhance it, one has to resort again to the special case I (section 5).

It is known, however (see, e.g., [42]), that also the asymmetries generated by N_2 and/or N_3 can survive if 1) they are produced non-thermally at reheating and 2) N_1 is not in thermal equilibrium at the reheating temperature T_{RH}. In fact, the asymmetries $\epsilon_{2,3}$ can be large (they are of the order of $m^2_{c,t}/(16\pi v^2)$ in the generic case and even larger in the special case II: $\epsilon_{2,3} \sim m_c/m_t$). However, partial thermalization of $N_{2,3}$ and subsequent washout can occur after reheating. Moreover, N_1 should not enter into thermal equilibrium at any temperature $T \lesssim T_{RH}$.

In this case an accurate computation of the final asymmetry would require to solve the complete set Boltzmann equations describing the evolution of the number densities of all three RH neutrinos and of $B - L$.

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References

[1] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, in proceedings of the *Workshop on the Unified Therory and Baryon Number in the Universe*, O. Sawada and A. Sugamoto eds., KEK report 79-18, 1979, p. 95, Tsukuba, Japan;

S.L. Glashow, *The future of elementary particle physics*, in *Cargese 1979, Proceedings, Quarks and Leptons*, pp. 687–713;
M. Gell-Mann, P. Ramond, and R. Slansky, *Complex spinors and unified theories*, in *Supergravity*, P. van Nieuwenhuizen and D.Z. Freedman eds., North Holland, Amsterdam 1979, p. 315;
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[2] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.

[3] A.S. Joshipura, E.A. Paschos and W. Rodejohann, *A simple connection between neutrino oscillation and leptogenesis*, *J. High Energy Phys.* **08** (2001) 029 [hep-ph/0105175];
M. Hirsch and S.F. King, *Leptogenesis with single right-handed neutrino dominance*, *Phys. Rev.* **D 64** (2001) 113005 [hep-ph/0107014];
T. Hambye, *Leptogenesis at the TeV scale*, *Nucl. Phys.* **B 633** (2002) 171 [hep-ph/0111089];
J.R. Ellis and M. Raidal, *Leptogenesis and the violation of lepton number and CP at low energies*, *Nucl. Phys.* **B 643** (2002) 229 [hep-ph/0206174];
J.R. Ellis, M. Raidal and T. Yanagida, *Observable consequences of partially degenerate leptogenesis*, *Phys. Lett.* **B 546** (2002) 228 [hep-ph/0206300];
L. Boubekeur, *Leptogenesis at low scale*, hep-ph/0208003;
G.C. Branco et al., *Minimal scenarios for leptogenesis and CP-violation*, *Phys. Rev.* **D 67** (2003) 073025 [hep-ph/0211001].

[4] F. Vissani and A.Y. Smirnov, *Neutrino masses and b-τ unification in the supersymmetric Standard Model*, *Phys. Lett.* **B 341** (1994) 173 [hep-ph/9405399];
A. Brignole, H. Murayama and R. Rattazzi, *Upper bound on hot dark matter density from* SO(10) *Yukawa unification*, *Phys. Lett.* **B 335** (1994) 345 [hep-ph/9406397].

[5] F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961;
S. Lavignac, I. Masina and C.A. Savoy, *$\tau \to \mu\gamma$ and $\mu \to e\gamma$ as probes of neutrino mass models*, *Phys. Lett.* **B 520** (2001) 269 [hep-ph/0106245].

[6] S. Davidson, *From weak-scale observables to leptogenesis*, *J. High Energy Phys.* **03** (2003) 037 [hep-ph/0302075].

[7] A.Y. Smirnov, *Seesaw enhancement of lepton mixing*, *Phys. Rev.* **D 48** (1993) 3264 [hep-ph/9304205].

[8] M. Tanimoto, *Seesaw enhancement of neutrino mixing due to the right-handed phases*, *Phys. Lett.* **B 345** (1995) 477 [hep-ph/9503318];
G. Altarelli, F. Feruglio and I. Masina, *Large neutrino mixing from small quark and lepton mixings*, *Phys. Lett.* **B 472** (2000) 382 [hep-ph/9907532];
T.K. Kuo, G.-H. Wu and S.W. Mansour, *Mass hierarchies and the seesaw neutrino mixing*, *Phys. Rev.* **D 61** (2000) 111301 [hep-ph/9912366];
T.K. Kuo, S.-H. Chiu and G.-H. Wu, *Neutrino mixing in the seesaw model*, *Eur. Phys. J.* **C 21** (2001) 281 [hep-ph/0011058];
M. Patgiri and N.N. Singh, *Right-handed Majorana neutrino mass matrices for generating bimaximal mixings in degenerate and inverted models of neutrinos*, *Int. J. Mod. Phys.* **A 18** (2003) 743 [hep-ph/0301254].

[9] K.S. Babu and S.M. Barr, *Large neutrino mixing angles in unified theories*, *Phys. Lett.* **B 381** (1996) 202 [hep-ph/9511446];
S.F. King, *Atmospheric and solar neutrinos with a heavy singlet*, *Phys. Lett.* **B 439** (1998) 350 [hep-ph/9806440]; *Large mixing angle msw and atmospheric neutrinos from single right-handed neutrino dominance and* U(1) *family symmetry*, *Nucl. Phys.* **B 576** (2000) 85 [hep-ph/9912492];
S.M. Barr and I. Dorsner, *A general classification of three neutrino models and* U_{e3}, *Nucl. Phys.* **B 585** (2000) 79 [hep-ph/0003058].

[10] M.S. Berger and B. Brahmachari, *Leptogenesis and yukawa textures*, *Phys. Rev.* **D 60** (1999) 073009 [hep-ph/9903406].

[11] D. Falcone and F. Tramontano, *Leptogenesis and neutrino parameters*, *Phys. Rev.* **D 63** (2001) 073007 [hep-ph/0011053];
D. Falcone, *Lepton number and lepton flavor violations in seesaw models*, *Mod. Phys. Lett.* **A 17** (2002) 2467 [hep-ph/0207308].

[12] E. Nezri and J. Orloff, *Neutrino oscillations vs. leptogenesis in* SO(10) *models*, *J. High Energy Phys.* **04** (2003) 020 [hep-ph/0004227].

[13] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and M.N. Rebelo, *Leptogenesis, CP-violation and neutrino data: what can we learn?*, *Nucl. Phys.* **B 640** (2002) 202 [hep-ph/0202030].

[14] D. Falcone, *Seesaw mechanism, baryon asymmetry and neutrinoless double* β *decay*, *Phys. Rev.* **D 66** (2002) 053001 [hep-ph/0204335];
D. Falcone, *Inverting the seesaw formula*, *Phys. Rev.* **D 68** (2003) 033002 [hep-ph/0305229].

[15] B. Pontecorvo, *Mesonium and antimesonium*, *Sov. Phys. JETP* **6** (1957) 429;
B. Pontecorvo, *Inverse beta processes and nonconservation of lepton charge*, *Sov. Phys. JETP* **7** (1958) 172;
Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.

[16] SUPER-KAMIOKANDE collaboration, T. Nakaya, *Atmospheric and long baseline neutrino*, *eConf* **C020620** (2002) SAAT01 [hep-ex/0209036];
SUPER-KAMIOKANDE collaboration, S. Fukuda et al., *Determination of solar neutrino oscillation parameters using 1496 days of Super-Kamiokande-I data*, *Phys. Lett.* **B 539** (2002) 179 [hep-ex/0205075];
SNO collaboration, Q.R. Ahmad et al., *Measurement of day and night neutrino energy spectra at SNO and constraints on neutrino mixing parameters*, *Phys. Rev. Lett.* **89** (2002) 011302 [nucl-ex/0204009];
KAMLAND collaboration, K. Eguchi et al., *First results from KamLAND: evidence for reactor anti-neutrino disappearance*, *Phys. Rev. Lett.* **90** (2003) 021802 [hep-ex/0212021];
CHOOZ collaboration, M. Apollonio et al., *Limits on neutrino oscillations from the CHOOZ experiment*, *Phys. Lett.* **B 466** (1999) 415 [hep-ex/9907037].

[17] H.V. Klapdor-Kleingrothaus et al., *Latest results from the Heidelberg-Moscow double-beta-decay experiment*, *Eur. Phys. J.* **A12** (2001) 147 [hep-ph/0103062];
IGEX collaboration, C.E. Aalseth et al., *The IGEX* ^{76}Ge *neutrinoless double-beta decay experiment: prospects for next generation experiments*, *Phys. Rev.* **D 65** (2002) 092007 [hep-ex/0202026].

JHEP09(2003)021

[18] J. Bonn et al., *The Mainz neutrino mass experiment*, *Nucl. Phys.* **91** *(Proc. Suppl.)* (2001) 273;
V.M. Lobashev et al., *Direct search for neutrino mass and anomaly in the tritium beta-spectrum: status of 'Troitsk neutrino mass' experiment*, *Nucl. Phys.* **91** *(Proc. Suppl.)* (2001) 280.

[19] D.N. Spergel et al., *First year Wilkinson microwave anisotropy probe (WMAP) observations: determination of cosmological parameters*, `astro-ph/0302209`.

[20] S. Hannestad, *Neutrino masses and the number of neutrino species from WMAP and 2DFGRS*, *J. Cosmol. Astropart. Phys.* **0305** (2003) 004 [`astro-ph/0303076`].

[21] E.K. Akhmedov, *Small entries of neutrino mass matrices*, *Phys. Lett.* **B 467** (1999) 95 [`hep-ph/9909217`].

[22] M. Frigerio and A.Y. Smirnov, *Structure of neutrino mass matrix and CP-violation*, *Nucl. Phys.* **B 640** (2002) 233 [`hep-ph/0202247`].

[23] M. Frigerio and A.Y. Smirnov, *Neutrino mass matrix: inverted hierarchy and CP-violation*, *Phys. Rev.* **D 67** (2003) 013007 [`hep-ph/0207366`].

[24] P.H. Frampton, S.L. Glashow and D. Marfatia, *Zeroes of the neutrino mass matrix*, *Phys. Lett.* **B 536** (2002) 79 [`hep-ph/0201008`];
Z. Xing, *Texture zeros and majorana phases of the neutrino mass matrix*, *Phys. Lett.* **B 530** (2002) 159 [`hep-ph/0201151`].

[25] A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, *See-saw realization of the texture zeros in the neutrino mass matrix*, *Phys. Lett.* **B 538** (2002) 96 [`hep-ph/0204291`].

[26] H. Fusaoka and Y. Koide, *Updated estimate of running quark masses*, *Phys. Rev.* **D 57** (1998) 3986 [`hep-ph/9712201`].

[27] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *On the anomalous electroweak baryon number nonconservation in the early universe*, *Phys. Lett.* **B 155** (1985) 36.

[28] M.A. Luty, *Baryogenesis via leptogenesis*, *Phys. Rev.* **D 45** (1992) 455;
M. Flanz, E.A. Paschos and U. Sarkar, *Baryogenesis from a lepton asymmetric universe*, *Phys. Lett.* **B 345** (1995) 248 [`hep-ph/9411366`];
M. Plumacher, *Baryogenesis and lepton number violation*, *Z. Physik* **C 74** (1997) 549 [`hep-ph/9604229`];
L. Covi, E. Roulet and F. Vissani, *CP-violating decays in leptogenesis scenarios*, *Phys. Lett.* **B 384** (1996) 169 [`hep-ph/9605319`];
W. Buchmuller and M. Plumacher, *CP asymmetry in Majorana neutrino decays*, *Phys. Lett.* **B 431** (1998) 354 [`hep-ph/9710460`].

[29] A. Pilaftsis, *Heavy majorana neutrinos and baryogenesis*, *Int. J. Mod. Phys.* **A 14** (1999) 1811 [`hep-ph/9812256`].

[30] W. Buchmuller, P. Di Bari and M. Plumacher, *Cosmic microwave background, matter-antimatter asymmetry and neutrino masses*, *Nucl. Phys.* **B 643** (2002) 367 [`hep-ph/0205349`].

[31] E.W. Kolb and M.S. Turner, *The early universe*, Addison-Wesley 1990;
H.B. Nielsen and Y. Takanishi, *Baryogenesis via lepton number violation in anti-GUT model*, *Phys. Lett.* **B 507** (2001) 241 [`hep-ph/0101307`].

JHEP09(2003)021

[32] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett.* **B 535** (2002) 25 [hep-ph/0202239].

[33] W. Buchmuller, P. Di Bari and M. Plumacher, *The neutrino mass window for baryogenesis*, *Nucl. Phys.* **B 665** (2003) 445 [hep-ph/0302092].

[34] H.V. Klapdor-Kleingrothaus, *Genius: a new underground observatory for non-accelerator particle physics*, *Nucl. Phys.* **110** *(Proc. Suppl.)* (2002) 364 [hep-ph/0206249].

[35] R. Gonzalez Felipe and F.R. Joaquim, *Is right-handed neutrino degeneracy compatible with the solar and atmospheric neutrino data?*, *J. High Energy Phys.* **09** (2001) 015 [hep-ph/0106226].

[36] S.L. Glashow, *Neutrinos with seesaw masses and suppressed interactions*, hep-ph/0301250.

[37] P.H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw neutrino masses*, *Phys. Lett.* **B 316** (1993) 312 [hep-ph/9306333];
K.S. Babu, C.N. Leung and J. Pantaleone, *Renormalization of the neutrino mass operator*, *Phys. Lett.* **B 319** (1993) 191 [hep-ph/9309223];
S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass operator renormalization revisited*, *Phys. Lett.* **B 519** (2001) 238 [hep-ph/0108005];
M. Frigerio and A.Y. Smirnov, *Radiative corrections to neutrino mass matrix in the Standard Model and beyond*, *J. High Energy Phys.* **02** (2003) 004 [hep-ph/0212263];
S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Running neutrino masses, mixings and CP phases: analytical results and phenomenological consequences*, hep-ph/0305273.

[38] N. Haba, N. Okamura and M. Sugiura, *The renormalization group analysis of the large lepton flavor mixing and the neutrino mass*, *Prog. Theor. Phys.* **103** (2000) 367 [hep-ph/9810471];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Nearly degenerate neutrinos, supersymmetry and radiative corrections*, *Nucl. Phys.* **B 569** (2000) 82 [hep-ph/9905381];
S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass matrix running for non-degenerate see-saw scales*, *Phys. Lett.* **B 538** (2002) 87 [hep-ph/0203233].

[39] R.N. Mohapatra and G. Senjanovic, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165;
J. Schechter and J.W.F. Valle, *Neutrino masses in* $\mathrm{SU}(2) \times \mathrm{U}(1)$ *theories*, *Phys. Rev.* **D 22** (1980) 2227;
C. Wetterich, *Neutrino masses and the scale of B-L violation*, *Nucl. Phys.* **B 187** (1981) 343;
G.B. Gelmini and M. Roncadelli, *Left-handed neutrino mass scale and spontaneously broken lepton number*, *Phys. Lett.* **B 99** (1981) 411;
E. Ma and U. Sarkar, *Neutrino masses and leptogenesis with heavy Higgs triplets*, *Phys. Rev. Lett.* **80** (1998) 5716 [hep-ph/9802445].

[40] H. Murayama and T. Yanagida, *Leptogenesis in supersymmetric standard model with right-handed neutrino*, *Phys. Lett.* **B 322** (1994) 349 [hep-ph/9310297];
G.F. Giudice, M. Peloso, A. Riotto and I. Tkachev, *Production of massive fermions at preheating and leptogenesis*, *J. High Energy Phys.* **08** (1999) 014 [hep-ph/9905242];
M. Fujii, K. Hamaguchi and T. Yanagida, *Leptogenesis with almost degenerate Majorana neutrinos*, *Phys. Rev.* **D 65** (2002) 115012 [hep-ph/0202210];
L. Boubekeur, S. Davidson, M. Peloso and L. Sorbo, *Leptogenesis and rescattering in supersymmetric models*, *Phys. Rev.* **D 67** (2003) 043515 [hep-ph/0209256].

[41] J.R. Ellis, M. Raidal and T. Yanagida, *Sneutrino inflation in the light of WMAP: reheating, leptogenesis and flavor-violating lepton decays*, hep-ph/0303242.

JHEP09(2003)021

[42] T. Asaka, H.B. Nielsen and Y. Takanishi, *Non-thermal leptogenesis from the heavier Majorana neutrinos*, *Nucl. Phys.* **B 647** (2002) 252 [hep-ph/0207023].